Linear Functions

Section 1: Introduction to Functions

Functions are fundamental to further study of mathematics or science. The idea of a function is best approached by looking at some examples.

1. The cost of renting a car is a function of the number of miles it is driven.

2. The pressure in a car tire is a function of the temperature of the tire.

3. The area of a square is a function of the length of a side.

4. The number of gallons of paint needed to paint a house is a function of the size of the house.

From these examples you can see that functions are quite simple, they consist of an **input**, a **rule** that transforms the input, and **output** results.

input ⟶ [rule] ⟶ output

Think of the formula for the area of a square, $A = s^2$. This represents a function. Someone inputs 2, the length of the side of a square, the rule is square the input number , and the area is output.

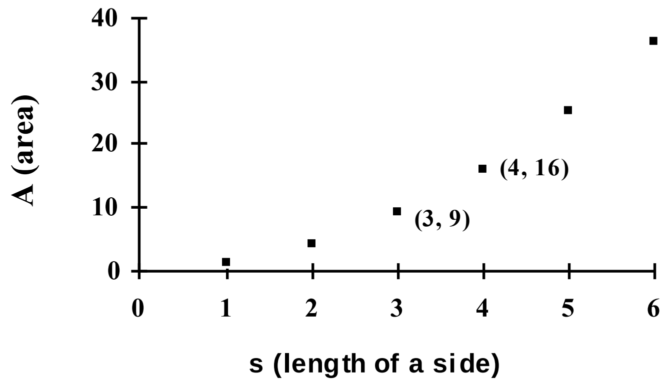
$2 \longrightarrow s^2 \longrightarrow 4.$

The set of all possible inputs is called the **domain** of the function and the set that consists of all possible outputs is called the **range** of the function.

Depending on the situation, a function can be best described as a **formula**, **data in a table, or as points on a graph.** When we looked at the formula for the area of a square, $A = s^2$, we were using an **expression or formula** to represent a function. We could also put the data in an **input-output** table.

Length of side s **(Input)**	1	2	3	4	...	
Area A **(output)**	1	4	9	16	...	

We can also write the data in the table as **ordered pairs**. The ordered pairs in this example would be (1, 1), (2, 4), (3, 9), and (4, 16). The first component in the pair is the input variable and the second component is the output variable. These ordered pairs are plotted on the next page. Data represented this way is said to be plotted on the **Cartesian plane**.



In mathematics, the meaning of a **function** is precise. It is a relationship between two quantities. When one quantity **uniquely** determines a second quantity, then the second quantity is a function of the first. The area of a square is a function of the length of a side because for any length of a side there is a **unique** area.

Activity 1

1. Not all functions have to mathematical. Which of the following represent functions? If it does not represent a function state why not.

   a) If you input a person's name, you get output the number of letters in their name.

   b) If you input a person, you get output the make of their car.

   c) If you input a person, you get output their social security number.

   d) If you input a set, you get output the number of elements in the set.

2. Functions are introduced in about the third grade with problems like the following:

   Use the table to help find each secret rule. Finish the table and write the rule in words and as a formula.

   a)

In	3	4	5	7	10
Out	7	8	9		

Rule: ______________________________

b)

In	5	6	8	10	15
Out	14	15	17		

Rule: ______________________________

____

Example 1 : Is y a function of x in the following relation?

x (in)	2	2	4	4	5
y (out)	3	1	5	3	7

Solution: For a relation to be a function, each input has to have a unique output. In this relation when a 2 is input the output is 3 and 1. Since the output is **not** unique for the input 2, this relation is **not a function.**

Activity 2

Is y a function of x in the following relation? Explain.

x (in)	3	4	5	7	10
y (Out)	10	10	10	10	10

____

Activity 3

x (in)	2	9	14	17
y (out)	0	7	12	15

Fill in the rule for the function given by the input-output table above.

x (input) → [ ] → y (output)

Write the function as a formula.

y = ________________.

Name the domain and the range of the function.

Section 2: Function Notation

The following table gives the maximum daily drug dosage of Ampicillin for children who weigh less than 10 kilograms (about 22 pounds).

Daily Ampicillin Dosage by Weight

W Child's Weight	D Daily Maximum Dosage
0	0
1	50
2	100
3	150
4	200
5	250
6	300
7	350
8	400
9	450
10	500

Source: Math for Meds: Dosages and Solutions, 6th ed. (San Diego: W.I. Publications, 1990): p. 198.

Weight and dosage are called **quantitative variables**. We say that

"dosage depends on weight" or "dosage is a function of weight."

Because dosage depends on weight, we can think of weight as the **input variable** and dosage as the **output variable**.

According to the table, the dosage for a child weighing 4 kilograms is 200 milligrams of Ampicillin. Mathematicians use a shorthand notation for this statement and write it as, D(4) = 200. (Read D(4) = 200 as "**D of 4 equals 200**.") This notation indicates a relation in which one variable is a function of another. When you see D(4) = 200, you can also think of this as when you input a weight of 4 you get a dosage of 200 milligrams for output. You could also write this as the ordered pair (4, 200).

Activity 4

Use the table above to answer the following questions.

1. Find the following:
   a) D(9) = _____ b) D(1) = _____

2. The maximum safe dosage for a child weighing 10 kg is 500 mg.
   Write this statement using function notation.

The following may help give more meaning to function notation.

$$D(4) = 200$$

↑ Name of function ↑ input ↑ output

Example 2 : A function is defined by the formula $f(x) = 4x - 1$. Find the following.

a) $f(1)$ b) $f(3)$ c) x so that $f(x) = 23$.

Solution: The function $f(x) = 4x - 1$ can be viewed as

x (input) ⟶ [multiply by 4 then subtract 1 from the result] ⟶ output

a) To find f(1) we input 1, multiply by 4 to get 4 and then subtract 1 to get 3. So, $f(1) = 3$.

A more compact way to write this is

$$\boxed{f(1) = 4 \cdot 1 - 1 = 3}\ .$$

Everyplace we had an x in $4x - 1$, we replaced it with a 1.

b) $\boxed{f(3) = 4 \cdot 3 - 1 = 12 - 1 = 11}$

c) This problem is asking you what you would input for x to get an output of 23. In other words, solve the following:

$$4x - 1 = 23.$$

Solving this equation, we get

$4x - 1 = 23$ *Add 1 to both sides.*

$4x = 24$ *Divide both sides by 4.*

$\boxed{x = 6}$

Therefore, when $x = 6$, $f(x) = 23$.

Activity 5

Given the following function rules, find f(3) and f(5).

a) $f(x) = x + 9$

f(3) = ____________________

f(5) = ____________________

b) $f(x) = x^2 - 4x + 7$

f(3) = ____________________

f(5) = ____________________

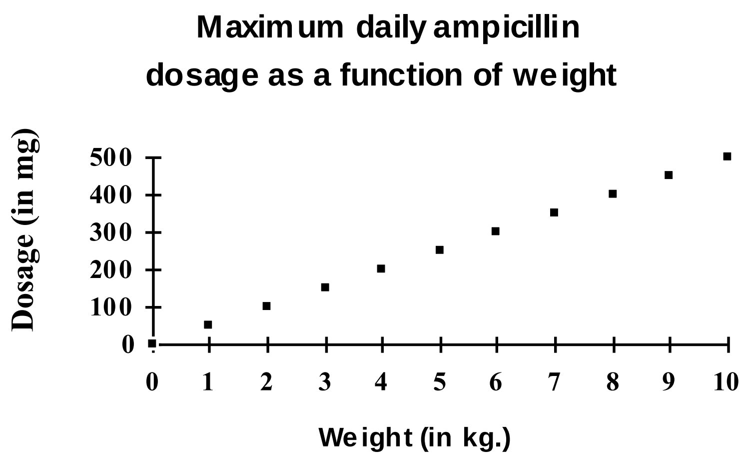
Section 3: Representing a Function Numerically, Graphically, and Algebraically

We saw that we can represent a function with a table. The maximum safe dosage of Ampicillin was a function of the child's weight and was represented by the following table.

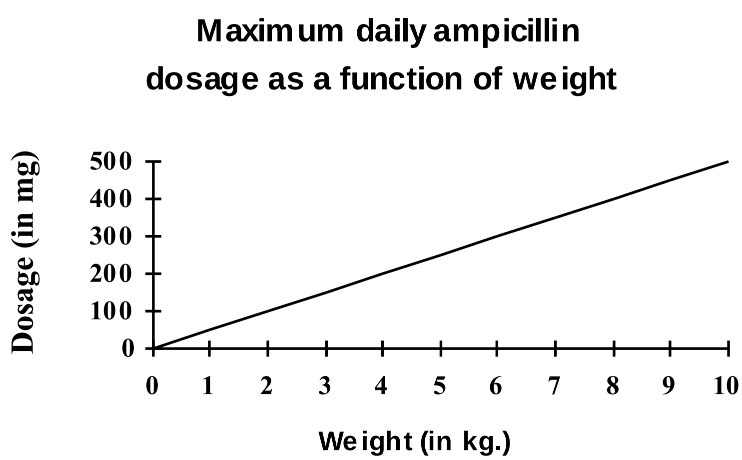
Daily Ampicillin Dosage by Weight

W Child's Weight	D Daily Maximum Dosage
0	0
1	50
2	100
3	150
4	200
5	250
6	300
7	350
8	400
9	450
10	500

Another way to represent a function is with a graph. The table above is graphed below.



We only plotted eleven points for the given function, but the domain of this function is all numbers between 0 and 10. If you plotted a point for all the values in the domain, the graph would look like the following.



Activity 6

With the help of the graph above, approximate the value D(3.5). (*What do you get for output when you input a weight of 3.5 kg?*)

Once data is collected and graphed an important task is to find a formula that best represents the data. Looking back at our table, one trend we notice is that the dosage is 50 times the weight. So we can now write the function as the following formula:

$$D = 50W.$$

Using our earlier notation, we can think of this formula in the following way:

Weight (input) → [Multiply by 50 (Rule)] → Dosage (output)

Multiply 3.5 by 50 to see how close your approximation was in Activity 6.

Activity 7

The approximate temperature at various depths is given by the function

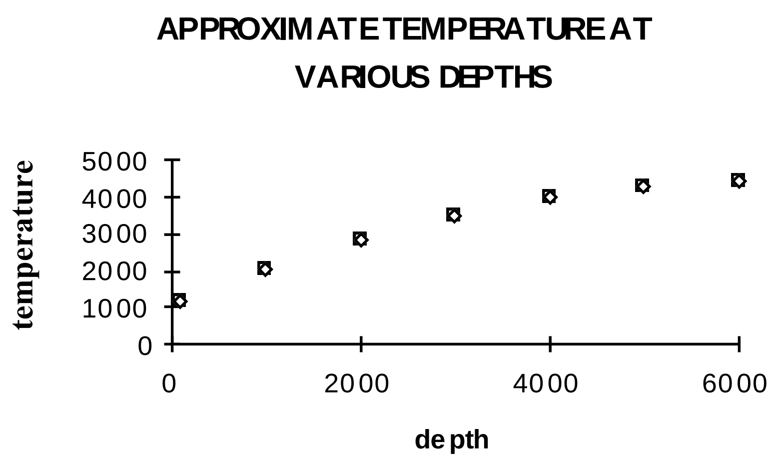
$$T(d) = (-\,8.255 \times 10^{-5})d^2 + 1.05d + 1110$$

where d is the depth in kilometers and temperature is □C. This function gives reasonable approximations to the earth's internal temperature for depths greater than 100 km. The function could be described numerically by the following table.

a) Find the missing entries.

Depth (km)	Temperature (□C)
100	____
1000	____
2000	____
3000	____
4000	____
5000	____
6000	____

Another way to represent a function is with a graph. The table you completed above is graphed below.

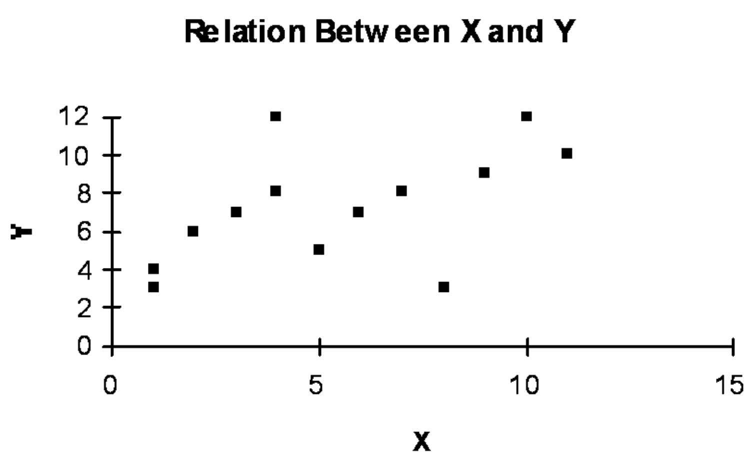


b) We only plotted seven points for the given function, but what is the domain of this function? If you plotted a point for all the values in the domain, what do you think the graph would look like?

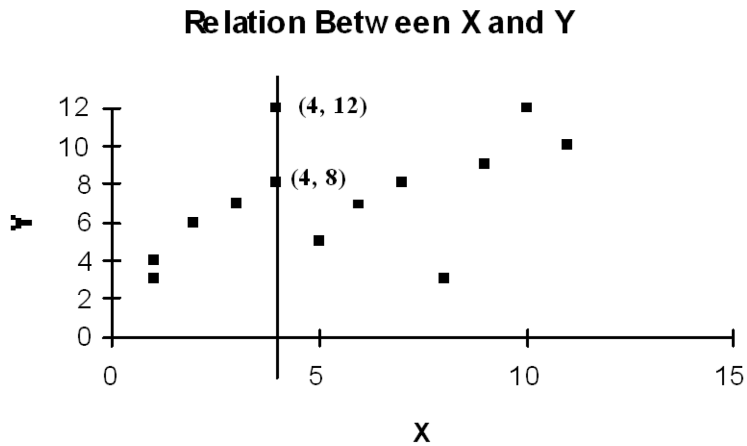
c) With the help of the graph above, approximate the value T(1500).

Section 4: Vertical Line Test

Example 3 : Not all graphs represent functions. Look at the definition of a function again and determine if the following graph represents a function.



Solution: Remember, for a relationship to be a function one quantity **uniquely** determines a second quantity. So the key question we are trying to answer is, " when you input a value of x, do you ever get more than one output?" Looking at the graph below, you can see that you have two ordered pairs, (4, 12) and (4, 8). This is saying that when you input 4 you get two different outputs. So the relation is **not** a function.



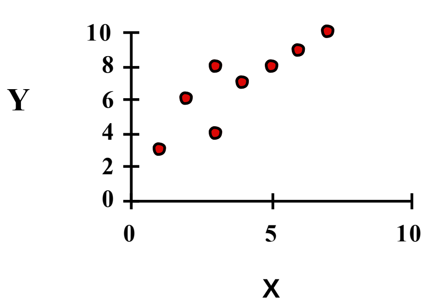
You can see from the last example, that if a vertical line hits the graph in more than one point, you do not have a function.

Vertical line test

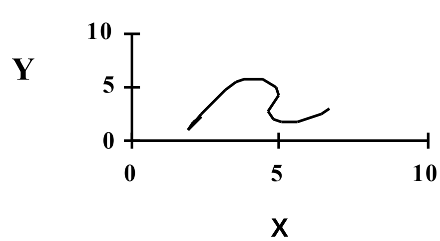
A graph represents a function if and only if every vertical line in the xy-plane intersects the graph in at most one point.

Activity 8

State whether the following relations represent a function.



a)



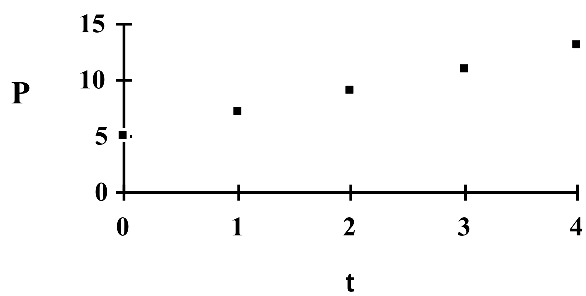
b)

Section 5: Linear Functions

Linear functions arise quite naturally in "real life." These are functions that represent a steady increase or a steady decrease. For example, one application in business is called **straight-line depreciation**. Under straight-line depreciation, the value of an item, such as an automobile is assumed to drop by the same amount each year. Look at the following input-output table where P is a function of t.

t (input)	0	1	2	3	4
P (output)	5	7	9	11	13

You see from the table that **P** is growing at the constant rate of 2 for each increase of 1 in **t**. If you graphed this data you would see that the points all lie on a straight line and if you found additional points based on the pattern in the table, you would see that the new points also lie on the same line.



In general:

A **linear function** describes a quantity that changes at a constant rate. The graph of a linear function is a straight line.

The federal income tax regulations for 1993 state that if you are single and your 1993 income was equal to or greater than $250,000, then your federal income tax is $79,772 plus 39.6 percent of the amount over $250,000. The tax for various incomes greater than $250,000 is given in the following table.

Income	**250000**	**251000**	**252000**	**253000**	**254000**	**255000**
Tax	**79772**	**80168**	**80564**	**80960**	**81356**	**81752**

Thinking of income as the **independent** (input) variable and tax as the **dependent** (output) variable, you can see that every increase of $1,000 in a person's income causes the tax to increase $396. It doesn't matter whether the income is $250000 or $400,000. The **rate of change** of tax with respect to income is a constant number, $\frac{396}{1000}$. It is the constant rate of change that makes a functional relationship linear. Thus tax is a **linear function** of income.

Activity 9

Graph the table below:

Income	**250000**	**251000**	**252000**	**253000**	**254000**	**255000**
Tax	**79772**	**80168**	**80564**	**80960**	**81356**	**81752**

If I is the income and T is the tax, we see that

$$\frac{\text{change in tax}}{\text{change in income}} = \frac{\Delta T}{\Delta I} = \frac{\$396}{\$1000} = \text{\$396 per \$1000 increase in income.}$$

Activity 10

Suppose you want to rent a van for a college club outing. You have two choices; renting from Keene State College or Keene Rent-a-Car. The rental rates are as follows:

Keene State College (KSC):

initial charge of $ 30 plus 40 cents per mile.

Keene Rent-a-Car (KRC):

initial charge of $ 20 plus 45 cents per mile.

a) From whom would you rent the van?

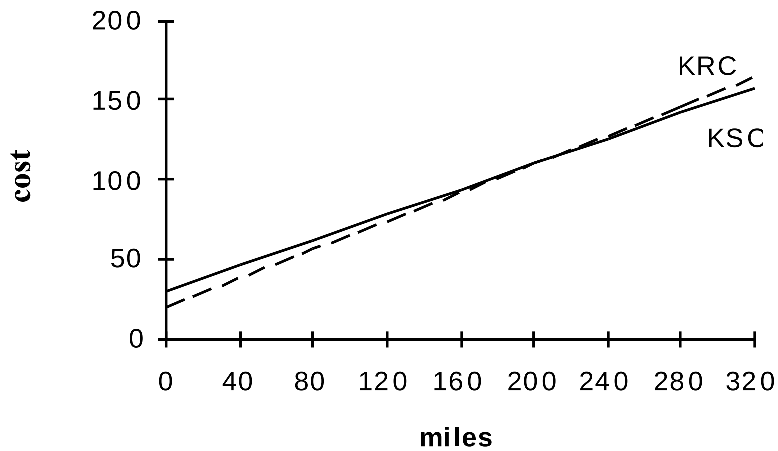
To analyze our problem we can start off by making a table.

b) Complete the following table.

Miles	0	40	80	120	160	200	240	280	320
KSC cost									
KRC cost									

c) What information about the relative costs do see from the table?

We can also use a graph to help us make a decision about who we should rent from. Both functions are plotted on the same coordinate system. What information can you get from the graph that you cannot get from the table above?



Section 6: Writing the Formula for a Linear Function

The process that we used to fill in the table above can help us write our functions as formulas. Look at our calculations for finding cells A, B, and C in the table.

Miles	0	40	80	120	160	200	240	280	320
KSC cost			A		B			C	

Cell A: **30 + 0.4(*80*)** = 30 + 32 = 62

Cell B: **30 + 0.4(*160*)** = 30 + 64 = 94

Cell C: **30 + 0.4(*280*)** = 30 + 112 = 142

Looking for a pattern in the three calculations, we could write this formula generally as

cost = 30 + 0.4(miles).

This is a little awkward, so let's just write the formula as

(1) **c = 30 + 0.4m** where **c** = cost and **m** = miles driven.

Make sure you understand the steps we used to arrive at formula (1) and then try the next problem.

Example 4 : Suppose that The Keene Record Store wants to print promotional flyers for its annual sidewalk sale. The store manager goes to a local printer and finds that there is a $20 setup cost and a 15 cent copying fee for each flyer produced. Determine the cost function, C, for the printing job.

Solution: We can start off by making sure we understand how this function works. Let's see if we can calculate the cost for a few different number of copies. If we only wanted 10 copies of the flyer, the cost would be

$$\mathbf{20 + .15(\underset{\text{\# of flyers}}{10}) = 21.50}$$

# of flyers
↓
20 + .15(10) = 21.50
↑ ↑ ↑
Fixed Cost/flyer Cost

If you wanted the cost to print 90 flyers the cost would be

$$\mathbf{20 + .15(90) = 33.50}$$

Looking at the pattern in the case of 10 flyers and 90 flyers, we can write the cost function as,

C = 20 + .15x, where **C** is the cost and
x is the number of flyers printed.

Written in more formal function notation we would have

$$\mathbf{C(x) = 20 + .15x}.$$

Activity 11

Suppose a case of fresh strawberries cost $10. Each day the strawberries sit in the store 50 cents is taken off the price. Write down a formula that gives the cost of the strawberries as a function of the number of days they sat in the store.

Start off by filling in the table looking at the calculations you performed for each cell.

days	0	3	8
cost			

C = ______________________

Linear Functions in General

A linear function has the form

y = f(x) = mx + b

and a graph where

m is the **slope**, or rate of change of y with respect to x
b is the **vertical intercept**, or value of y when x is zero.

The slope of a line characterizes the "steepness " of the line. It is simply a measure of how rapidly the height increases as we move from left to right along the line.

Slope

The **slope** of a line, denoted by m is a measurement of the steepness of the line. Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope of the line is computed by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line also gives the **average rate of change** of y with respect to x.

Example 5 : For the points (3, 7) and (-1, 2),

a) compute the average rate of change between the points.

b) If the independent variable increased by 1 unit, how will this affect the dependent variable?

Solution:

a) Using the definition of the average rate of change above we get

$$m = \frac{2-7}{-1-3} = \frac{-5}{-4} = \frac{1.25}{1}$$

b) Since the average rate of change is m = 1.25 and writing this as the ratio $\frac{1.25}{1}$, you can see that if x, the independent variable, increases by 1 unit then the dependent variable will increase by 1.25 units.

Note: All of the following are ways to interpret slope:

$$m = \text{ Average rate of change } = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in output}}{\text{change in input}}$$

$$= \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x}$$

Looking again at our example about renting a van from Keene State College, we have the cost function **c = 0.4m + 30**. Here our independent variable is m (miles) and the dependent variable is c (cost). The slope is the coefficient for the independent variable, in this case the slope is .4. As the definition above says, the slope is given by

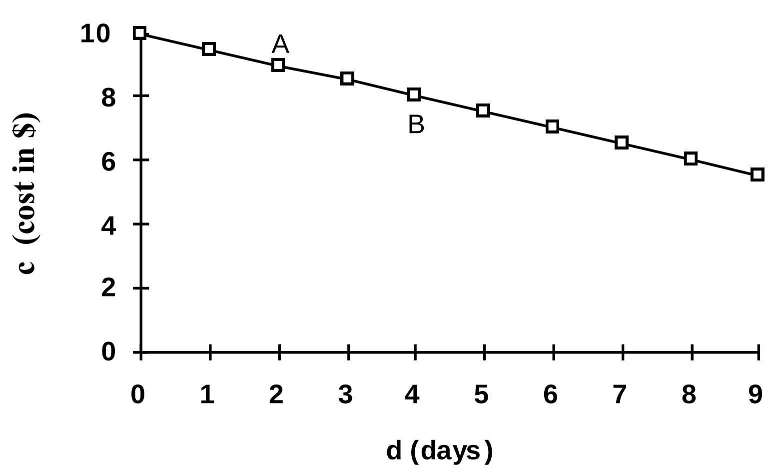
$$\textbf{slope} = \frac{\textbf{change in cost}}{\textbf{change in miles}} \ .$$

It is a ratio and you want to think of it that way. If the slope is .4, as a ratio it can be written as $\frac{.4}{1}$, which means every time the miles driven increases by 1 mile the price increases by \$ .4 or 40 cents.

In the last activity we wrote a cost function that was

c = 10 - .5d

where d equaled the number of days that strawberries sat in a store, and c was the cost. The graph of this function is given below.



The slope of the graph is **-.5**. (The function c = 10 - .5d is in the form **y = mx + b**, so the slope is the coefficient of the independent variable.)

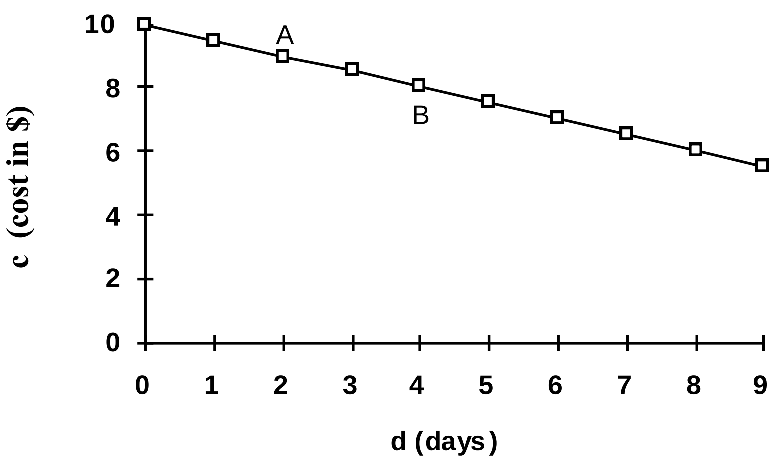
Example 6 : In the last example, the function was **c = 10 - .5d**. The slope is -.5 and remembering that the slope is a ratio, this can be written as $\frac{-.5}{1}$. This can be interpreted to mean that for every day the strawberries stay in the store, the price **decreases** by 50 cents.

Activity 12

Write the slope of the function below as a ratio and give an interpretation of the slope.

P(x) = -2.5x + 150, where x = units demanded for a product and
P = price charged for the product (in dollars)

Activity 13



a) What are the coordinates of point A and B on the graph above ?

A (__, __) B(__, __)

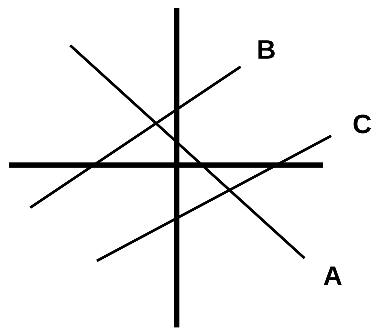
b) Once you know two points on the graph of a straight line you can get the slope by the definition of slope.

$$\textbf{slope (m)} = \frac{\textbf{change in dependent variable}}{\textbf{change in independent variable}}$$

Using points A and B we get m = $\frac{\_\_ - \_\_}{\_\_ - \_\_} = \frac{\_\_}{\_\_}$ = ____ .

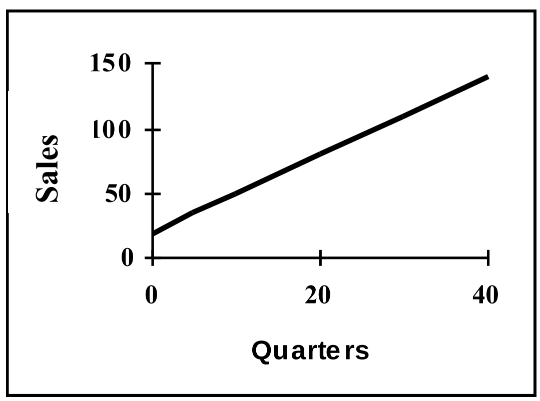
Note: When the slope of a linear function is negative the graph will be falling as you go from left to right. The opposite will be true when the slope is positive, the graph will be rising as you go from left to right.

Example 7 : In the figure below, lines B and C have positive slopes (they rise from left to right) and A has a negative slope (falls from left to right).



In addition, remember the slope measures the steepness of a line. If you take the absolute values of the slope, the larger the slope the steeper the line. Looking at lines B and C above, since B is a little steeper than C, it would have the larger slope.

Example 8 : Write a possible equation for the linear function below.



Solution: Remember, any linear function is in the form

y = f(x) = mx + b, where **m** is the slope and **b** is the vertical intercept.

In this example, the line crosses the vertical axis (sales) at about 20, so **b = 20**. To get the slope we have to approximate two points that lie on the line. The points (0, 20) and (10, 50) look like they lie on the line, so

$$\mathbf{m} = \frac{50-20}{10-0} = \frac{30}{10} = \mathbf{3}.$$

The equation of the line is then **Sales = 3Quarters + 20.**

This is a little awkward, so let's just write it as

S = 3Q + 20, where S is the sales and Q is the quarter.

Example 9 : Find the slope of the line joining the following pairs of points.

a) (5, 9) and (4, 1) b) (-5, 1) and (6, 8)

Solution: Remember the definition of the slope of a line:

$$\textbf{slope} = \frac{\textbf{change in dependent variable}}{\textbf{change in independent variable}}$$

a) $\mathbf{m} = \frac{9-1}{5-4} = \frac{8}{1} = 8$ or $m = \frac{1-9}{4-5} = \frac{-8}{-1} = 8$

b) $m = \frac{8-1}{6-(-5)} = \frac{7}{11}$ or $m = \frac{1-8}{-5-6} = \frac{-7}{-11} = \frac{7}{11}$

Activity 14

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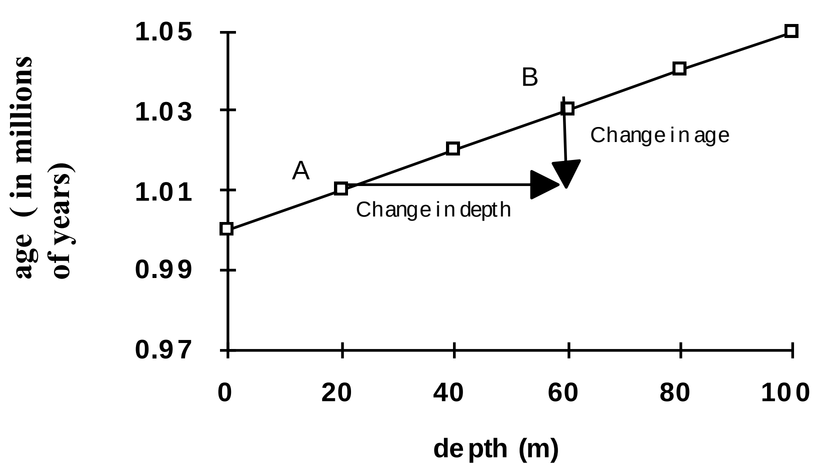
a) Find the slope of the following linear function using the definition. (Remember the slope is given by $m = \frac{\text{change in age}}{\text{change in depth}}$.) This particular function shows the relationship between ages of buried sediments in a dry lake bed to depth. Age is measured in millions years.

$$m = \frac{\_\_ - \_\_}{\_\_ - \_\_} =$$

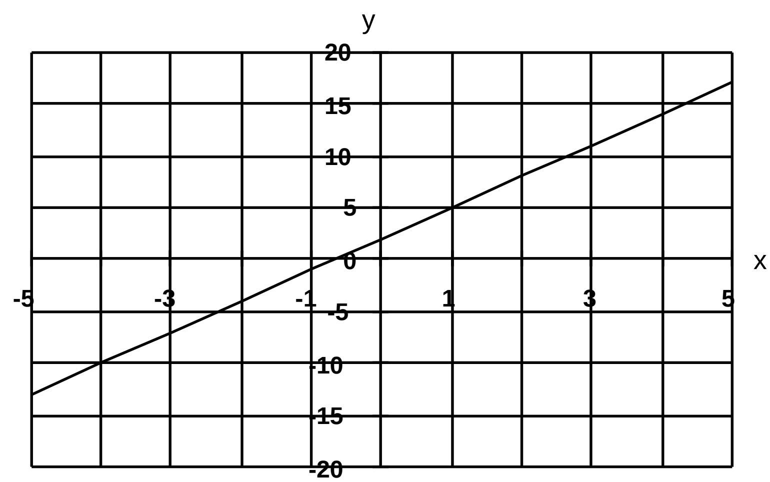


b) This is a linear function so it can be put in the form a = md + b, where m is the slope and b is the vertical intercept. For the graph above find the vertical intercept.

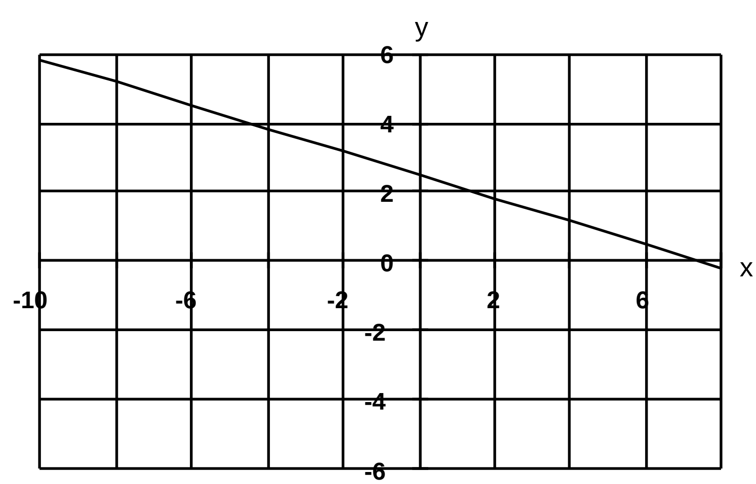
b = ______

c) Therefore the function can be written as a = __d + ___ .

d) Approximate the slopes of the following graphs.



Graph #1



Graph #2

Section 7: Break-Even Analysis

In break-even analysis the objective is to determine the break-even point. This is where the cost and revenue equal each other. The usual approach to finding the break-even point is to find the total cost function, C(x), and the total revenue function, R(x), where x represents the level of output. The break-even point can now be determined by setting R(x) equal to C(x) and solving for x.

Example 10 : Suppose the total revenue function is represented by

$$R(x) = 30x$$

and the total cost function is represented by

$$C(x) = 20x + 50.$$

Find the break-even point.

Solution: The break-even point occurs when total revenue equals total cost, or when

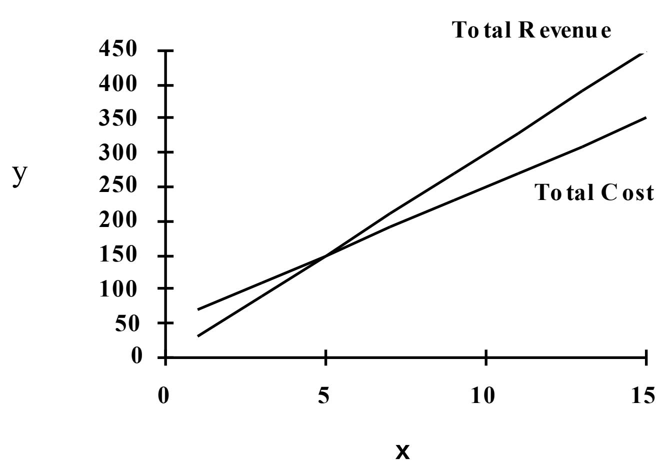
$$\mathbf{R(x) = C(x).}$$

So in this problem we compute the break-even point as follows:

$$30x = 20x + 50$$
$$10x = 50$$
$$x = \mathbf{5} \text{ units}$$

The break-even point is **5** units.

You can also solve the above example graphically. The revenue function and cost function are graphed on the same axis and the point where they intersect is the break-even point. You can see that you get the same solution, 5 units, as we computed above.



Exercises for Linear Functions

Do all the exercises on separate paper, showing all work neatly.

1. Given $D(t) = 2.45t^3 + 0.1t - 120$, find the following.

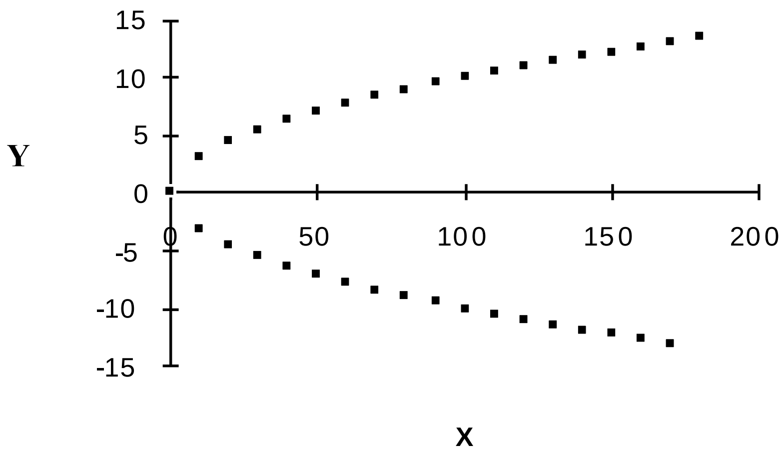
   a) D(5) b) D(4.2)

2. Given $f(x) = 3x^2$. The domain of this function is all real numbers, what is the range? Remember the range is your output values.

   Hint: No matter what value you input for x, what can you say about the output?

3. Explain why the volume of a sphere, $V = \frac{4}{3}\pi r^3$, is a function of the radius of a sphere. What is the domain of the function?

4. Explain why the following graph does not represent a function.



5. Without graphing determine if the following data represent a linear function? Explain your answer.

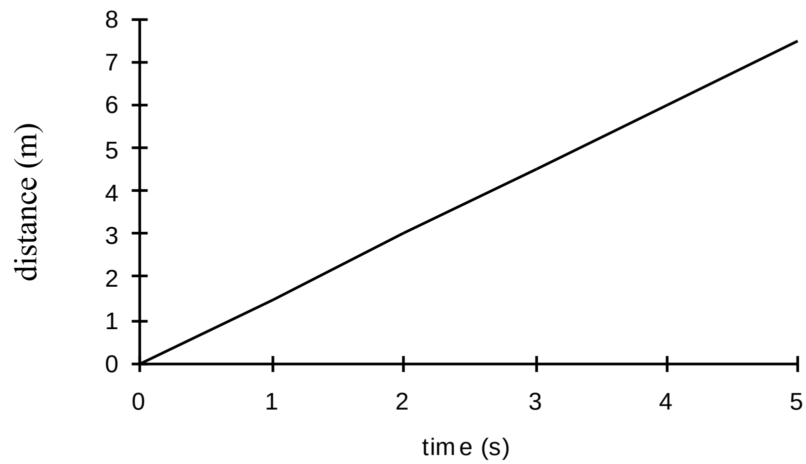
X	5.2	5.3	5.4	5.5	5.6
Y	27.8	29.2	30.6	32.0	33.4

   Hint: To recognize that a function given by a table of data is linear; look for differences in the dependent variables that are constant for equally spaced values of the independent variable.

6. Find the slope of the line through the given points.

a) (2, 5) and (6,18)   b) (-1, 5) and (3, -6)

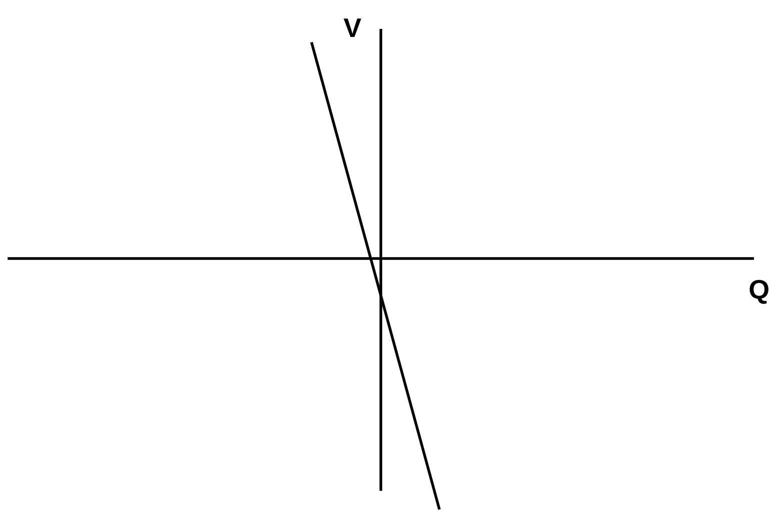
7. Given the following distance vs time graph, write a formula for the function.



8. Is y a function of x in the following relation? Explain.

x	**2**	**4**	**6**	**7**	**9**	**10**
y	**12**	**15**	**15**	**16**	**17**	**19**

9. Write a possible equation of the following function.

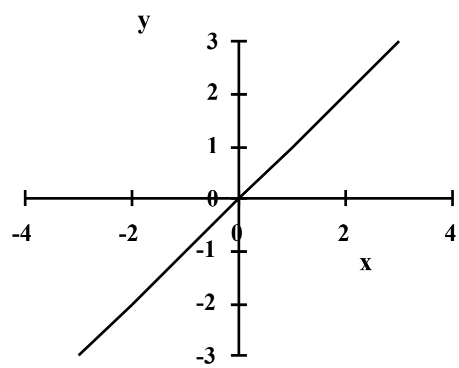


10. Let variables F and G be related by the equation F = 3G - 7.

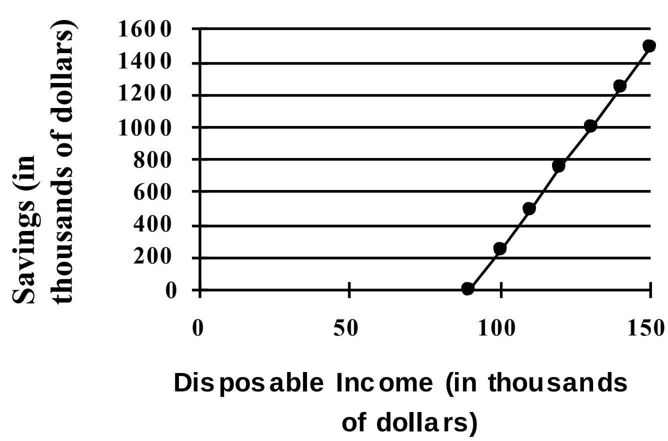
a) Determine the rate of change of F with respect to G.

b) If G changes by 8 units, how much does F change?

11. Why couldn't y = - x - 6 be the equation of the following line?



12. Compute the slope of the Savings vs. Disposable Income line below.



13. Which of the following lines is steeper?

a) C = -5x + 100 b) C = 2x +110 c) C = 4.5x

14. Suppose the total revenue function is represented by **R(x) = 75x** and the total cost function is represented by **C(x) = 2500 + 60x**. Find the break-even point.

LINEAR FUNCTIONS

Activity 1:

1. a) A function because each name as a unique number of letters.
EX/ Brian → 5
Sarah → 5
John → 4
Kim → 3

b) Not a function because each person may not own only one car.

EX/
Brian → Ford
Brian → Chevy
Brian → Volvo

c) A function because each person has only one social security number.

d) A function because each set will have a unique number of elements.

2. a) Table: 11, 14
Rule: words: add 4 to each input value
formula: $y = x + 4$

b) Table: 19, 24
Rule: words: add 9 to each input value
formula: $y = x + 9$

****

Activity 2:

Yes, y is a function of x because for every x there is exactly one y. If you graphed the points given in the table, you would get a horizontal line.

Note: If the x and y values were reversed, you would no longer have a function because the input of 10 would have many outputs. The graph would be a vertical line.

****

Activity 3:

rule: x(input) → [subtract 2] → y(output)

formula: $y = x - 2$

domain (input): {2, 9, 14, 17}

range (output): {0, 7, 12, 15}

****

Activity 4:

1. a) D(9) = 450 b) D(1) = 50 2. D(10) = 500

Activity 5:

a) $f(x) = x + 9$
$f(3) = 3 + 9 = 12$
$f(5) = 5 + 9 = 14$

b) $f(x) = x^2 - 4x + 7$
$f(3) = 3^2 - 4(3) + 7 = 9 - 12 + 7 = -3 + 7 = 4$
$f(5) = 5^2 - 4(5) + 7 = 25 - 20 + 7 = 5 + 7 = 12$

****
Activity 6:

D(3.5) = 175 In other words, the maximum safe dosage for a child weighing 3.5 kg is 175 mg.

****
Activity 7:

a)

Depth (km)	Temperature (^{0}C)
100	1214.2
1000	2077.5
2000	2879.8
3000	3517.1
4000	3989.2
5000	4296.3
6000	4438.2

b) The domain of this function is all numbers between 100 and 6000.

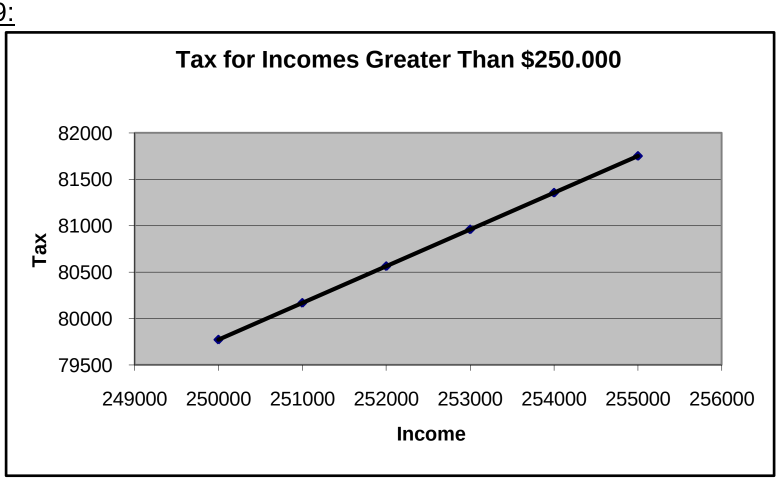
If you plotted a point for all the values in the domain, the graph would be smooth curve.

c) $T(1500) \approx 2500$ (check with formula: $T(1500) \approx 2499.3$)

****
Activity 8:

a) Fails the vertical line test – is not a function.
b) Fails the vertical line test – is not a function.

****
Activity 9:



Activity 10:

a) It depends on how many miles you plan to drive.

b)

MILES	0	40	80	120	160	200	240	280	320
KSC	30	46	62	78	94	110	126	142	158
KRC	20	38	56	74	92	110	128	146	164

c) At 200 miles, they both cost the same. KRC is lower until 200 miles, but KSC is lower after 200 miles.

****
Activity 11:

Days	0	3	8
Cost	10 – 0.50(0) = 10	10 - 0.50(3) = 8.5	10 – 0.50(8) = 6

$c = 10 - 0.50d$

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Activity 12:

The slope is $\frac{-2.5}{1}$. This can be interpreted to mean that for each unit of the product demanded, the price charged decreases by$2.50.

****
Activity 13:

a) A(2, 9) and B(4, 8)

b) m = $\frac{9-8}{2-4} = \frac{1}{-2} = -\cdot 5$

****
Activity 14:

a) A(20, 1.01) and B(60, 1.03)

m = $\frac{1.03-1.01}{60-20} = \frac{.02}{40} = \frac{2}{4000} = \frac{1}{2000}$

Therefore, for every 2000 m, the age increases 1 million years.

b) The vertical intercept is 1.

c) Therefore, the function can be written as a = $\frac{1}{2000}d + 1$ or $a = 0.0005d + 1$.

d) Graph #1: approximating the points A(1, 5) and B(-4, -10),

$$m = \frac{-10-5}{-4-1} = \frac{-15}{-5} = 3$$

Graph #2: approximating the points A(2, 1.9) and B(6, 0.5),

$$m = \frac{1.9-0.5}{2-6} = \frac{1.4}{-4} = -0.35$$