Solving Equations, Formulas, and Proportions

Section 1: Introduction

One of the basic goals of algebra is solving equations. An **equation** is a mathematical statement in which two expressions equal one another. We want to take our time looking at a variety of ways to solve equations. The process of solving equations should be understood and not just be a set of memorized rules. Manipulating expressions and equations is important because sometimes the form of an expression might be inappropriate for a particular task and you will have to simplify expressions or combine expressions to produce a new one.

In order to thoroughly cover this topic, we will have to review some topics such as simplifying algebraic expressions.

Section 2: Algebraic Expressions

A meaningful combination of numbers, symbols, and arithmetic operations is called an **algebraic expression**. Some examples of algebraic expressions are:

$$x, \quad 46, \quad 3x + 6, \quad n^3 + 7n + 19, \quad \text{and} \quad \frac{y - 5}{3}.$$

As was mentioned previously, when two expressions equal each other the statement is called an equation. Given an expression such as **$6 - 2(3x - 6)$,** it is important to be able to tell how many **terms** are in the expression and also be able to tell what they are. Remember **terms** are the parts of an expression that are separated by an addition or a subtraction sign. For example, $3x + 9$ has two terms, $3x$ and 9. If you were given an expression such as $3x - 9$, this also has two terms. Notice that $3x - 9$ is the same as
$3x + {}^-9$, so the two terms are $3x$ and ${}^-9$.

Example 1 : How many terms does the expression $3x^3 - 5x + 21$ have? Name them.

Solution: Since $3x^3 - 5x + 21 = 3x^3 + {}^-5x + 21$ there are three terms: $3x^3$, ${}^-5x$, and 21.

Note: Recall that the term ${}^-5x$ has two **factors**, ${}^-5$ and x.

Example 2 : How many terms does the expression

$3(x^2 + 3x + 1) + 4(x + 5)$ have? Name them.

Solution: There are two terms: $3(x^2 + 3x + 1)$ and $4(x + 5)$.

Each term in the last example has two factors.
$3(x^2 + 3x + 1)$ has factors 3 and $x^2 + 3x + 1$ and $4(x + 5)$ has factors 4 and $x+5$.

Activity 1

Write an algebraic expression that has three terms.

Take the expression you just wrote and multiply it by 4. How many terms does the resulting expression have?

Activity 2

Name the terms in the expression $6 - 2(3x - 6)$ and then simplify the expression.

Activity 3

Simplify $4\{3 + 5(x - 6)\}$ first by distributing the 4.

Section 3: Solutions to Equations

In algebra we are often interested in finding **solutions** or **roots** of an equation. Two equations are **equivalent** if they have exactly the same roots. There is a series of operations that will allow you to change an equation into an equivalent equation.

Operations for Changing Equations into Equivalent Equations

1. **Add or subtract the same number or algebraic expression to both sides of the equation.**

2. **Multiply or divide both sides by the same number.**
   Note: You cannot divide both sides by 0.

3. **Combine like terms on either side of the equation.**

4. **Interchange the two sides of the equation.**

It is important to note that you add or subtract terms from both sides and you multiply or divide both sides by factors.

The following equations are equivalent because each has a solution or root of 5.
Check this.

a) $4x = 20$ b) $3x - 2 = 13$ c) $x = 5$

d) $\frac{3x - 5}{2} = 2x - 5$ e) $-3 = 2 - x$

Of the five equations above, the easiest one to solve is c, since there is nothing to do. The solution is 5. Our goal when solving an equation algebraically is to transform the equation into a simpler but equivalent equation. The rules for accomplishing this are the operations above. For example, when we are asked to solve an equation like $\frac{2x - 5}{3} = 2x + 3$, we have to find an equivalent equation that is in the form x = "*something*". We accomplish this by using the four operations above.

Activity 4

Write three equations that are equivalent to $4x + 3 = 10 - 3x$.

Activity 5

Do the following problems without finding a solution to the given equations. In each case explain how you got your answer.

a) If $20x = \frac{1}{3}$, then $60x =$ _______.

b) If $8y = 0.025$, then $800y =$ _________.

c) If $7a = 10$, then $8a =$ _________.

Example 3 : Solve $3x + 27 = 6x$

Solution:

$3x + 27 = 6x$ *Our goal is to obtain an equation in the form x = something.*

$27 = 3x$ *Subtract 3x from both sides.*

$\frac{27}{3} = \frac{3x}{3}$ *Divide both sides by 3.*

$9 = x$ or $\boxed{x = 9}$ *Interchange the two sides of the equation.*

Now you should check the solution by substituting a 9 for x in the original equation.

Does $3(9) + 27 = 6(9)$?

Example 4 : Solve $7y + 6 = 216 - 3y$

Solution:

$7y + 6 = 216 - 3y$ *Our goal is to obtain an equation in the form y = something.*

$10y + 6 = 216$ *Add 3y to both sides.*

$10y = 210$ *Subtract 6 from both sides.*

$\boxed{y = 21}$ *Divide both sides by 10.*

Check the solution.

Contrary to what many people think, there is **not** only one "correct" way to solve a given equation. Do not try to memorize a separate method for each type of equation. In the next example we will solve an equation in a non-standard way to illustrate the fact that there are many ways to solve an equation.

Example 5 : Solve $3(2x - 6) = 24$

Solution: We will do this problem first by eliminating of the 3 from the left side of the equation. Three is a factor so we have to divide both sides by 3. Then to eliminate the 2 from the left side we have to divide both sides (every term) by 2.

$3(2x - 6) = 24$	
$2x - 6 = 8$	*Divide both sides by 3.*
$x - 3 = 4$	*Divide both sides (every term) by 2.*
$\boxed{x = 7}$	*Add 3 to both sides.*

Keep in mind that all four equations above are equivalent, which means that 7 has to be a solution in each one. Knowing this can come in handy if your solution does not check because you can go back and work backward and find the first equation that doesn't check. This will then tell you where an error occurred.

Activity 6

Solve the following equations.

a) $6(b - 2) + 12 = 30$

b) $8 + 4(5x - 1) = 10$

c) $6 - 2(3x - 2) = 7$

Activity 7

Solve 3(4x - 8) - 2(3x + 5) = 4(x + 7) in two different ways.

If an equation contains fractions, multiply both sides of the equation (which is the same as every term in the equation) by a common denominator.

Example 6 : Solve $\frac{n}{2} + 5 = \frac{2n}{3}$

Solution: $6(\frac{n}{2} + 5) = 6(\frac{2n}{3})$ *Multiply both sides by 6 since 6 is the common denominator of* $\frac{n}{2}$ *and* $\frac{2n}{3}$.

$\frac{6n}{2} + 6 \cdot 5 = \frac{12n}{3}$ *Distributive property*

3n + 30 = 4n

$\boxed{30 = n}$ *Subtract 3n from both sides*

Example 7 : Solve the equation $\frac{2x + 5}{5} - \frac{x - 4}{2} = 20.$

Solution : *Anytime you have more than one term in the numerator or denominator, use parentheses.*

$$\frac{(2x + 5)}{5} - \frac{(x - 4)}{2} = 20$$

A common denominator of the two fractions is 10. Multiply both sides (every term) by this number, then solve the resulting equation.

$$10\left[\frac{(2x + 5)}{5} - \frac{(x - 4)}{2} \right] = 10(20)$$

$$\frac{10(2x+5)}{5} - \frac{10(x-4)}{2} = 10(20)$$

$$2(2x + 5) - 5(x - 4) = 200$$

$$4x + 10 - 5x + 20 = 200$$

$$-x + 30 = 200$$

$$-x = 170$$

$$\boxed{x = -170}$$

Activity 8

Solve the following equation.

$$\frac{x + 5}{3} + 6 = \frac{x}{2}$$

Section 4: Checking Solutions

In solving equations, we can check to see if we obtained the correct solution. If we substitute the solution into the original equation, we must obtain a true solution.

Example 8 : Suppose you solved the equation 4.0x - 0.48 = 0.80x + 4.0 and obtained the solution x = 1.4. Use your calculator to check the solution.

Solution: Using a scientific calculator you would substitute 1.4 for x on the left-hand side and then on the right-hand side. You should get the same answer if 1.4 is the solution. Enter the following on your calculator:

[4] [x] [1.4] [-] [.48] [=] **5.12**

Now evaluating the right-hand side when x = 1.4, enter

[.8] [x] [1.4] [+] [4] [=] **5.12**

So the solution checks.

You could also use the TEST key to check your solution (TI 80, 81, 82, and 83). To check a solution to the equation, we have to determine if the following is true:

$$4.0(1.4) - 0.48 \stackrel{?}{=} 0.80(1.4) + 4.0$$

Enter 4.0(1.4) - 0.48 = 0.80(1.4) + 4.0 using the equals sign from [2nd] [TEST] . After you hit the [ENTER] key, if the equation is true, the calculator will return a 1. If the equation is false, the calculator will return a 0.

Note: ***If you were solving this equation, you could multiply both sides of the equation by 100 to eliminate the decimals.***

Activity 9

Check whether $x = \frac{32}{3}$ is a solution to the equation $\frac{1}{7}(x + 8) = \frac{2}{5}(x - 4)$.

Section 5: Formulas

A **formula** is an equation that contains more than one variable. Formulas appear in all fields from engineering to economics. For example, one formula used in geology is $\Delta \mathbf{J} = \mathbf{m}/\mathbf{W_d}$. Often it is necessary to manipulate the equation to solve for a particular symbol that appears in it. For example, in the formula above it might be necessary to solve for $\mathbf{W_d}$. The goal would be to get $\mathbf{W_d}$ by itself on one side of the equation. As you will see, we will accomplish this by the same steps we used in the previous sections to solve an equation. In addition to manipulating formulas to solve for a specific variable we will also be looking at formulas for number sequences.

The usefulness of manipulating formulas may not be apparent unless you have taken courses where you had to apply algebra. The following example will hopefully illustrate the usefulness of manipulation and something we will be doing more of called **algebraic reasoning.**

Example 9 : Given a circle and a square of the same perimeter, which will contain more area?

Solution: The perimeter or **circumference** of a circle is given by $2\pi r$, where r is the radius. Let p be the perimeter of the square. Since the circle and the square have the same perimeter, we have

(1) $$\mathbf{2\pi r = p}$$

Recall that the perimeter of a square is found by adding the four sides or by multiplying one side by 4, since the sides are equal.

If the square has a perimeter of p, then the sides have length $\frac{p}{4}$. Since the area of a square is found by squaring one side, we have

$$\left(\frac{p}{4}\right)^2 = \frac{p^2}{16}\ .$$

The area of the circle is πr^2. To answer the question, we have to determine which is larger, $\frac{p^2}{16}$ or πr^2. These are hard to compare because one formula is in terms of p and the other in terms of r. We have to try and get them in the same variable. To do this take equation (1) above and solve it for r.

$$r = \frac{p}{2\pi}$$

Substituting this value into πr^2 we get

$$\pi r^2 = \pi\left(\frac{p}{2\pi}\right)^2 = \pi \cdot \frac{p^2}{4\pi^2} = \frac{p^2}{4\pi}\ .$$

To finish the problem we have to compare $\frac{p^2}{16}$ (area of the square) with $\frac{p^2}{4\pi}$ (area of the circle) and determine which is larger. Recall from arithmetic that if two fractions have the same numerators and different denominators, the one with the smallest denominator is the larger fractions. Looking at some examples will convince you that this is true. So in our example since $4\pi > 16$, then $\frac{p^2}{16} < \frac{p^2}{\pi}$. Therefore, the area of the circle is greater than the area of the square.

There is a relationship in geology that says the age of sediment at the bottom of a lake equals its depth multiplied by a constant. This can be expressed by the formula,

Age = k · Depth, where **k** is a **constant**.

This formula tells us the age of sediment if we know the depth. Suppose we wanted a formula which tells us the depth we would need to dig to reach sediments of a specified age. In this case, we would have to manipulate the formula so it is in the form "**Depth = something**." Using our operations for solving equations, since **k** is a factor we divide both sides by **k**. This would give

$$\frac{\text{Age}}{\text{k}} = \text{Depth}$$

which can be rewritten as

$$\text{Depth} = \frac{\text{Age}}{\text{k}}.$$

This is the expression we wanted. It is very important in all the examples we do that you understand the logic. Remember that the symbols represent numbers and you should proceed as we did in the last sections.

Activity 10

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1) Manipulate **Age = k · Depth** to give an expression for k.

2) At a depth of 5m, a particular sediment is 4500 years. Find the sedimentation constant, k.

Example 10 : Suppose you had the more complex sediment formula

$$\mathbf{A = kD + A_t}$$

where **A** is age of sediment, **D** is the depth, and $\mathbf{A_t}$ is the age of the top. Solve this equation for **D**.

Solution: Our goal is to get D by itself on one side of the equation. So we have to get the k and A_t on the other side. It won't matter what you eliminate first, although the final answers might look different. Let us decide to eliminate A_t first.

$A = kD + A_t$

$A - A_t = kD$ *subtract A_t from both sides since A_t is a term*

$\frac{A - A_t}{k} = D$ *divide both sides by k since k is a factor*

$$\mathbf{D = \frac{A - A_t}{k}}$$

The answer in the last example could also have been written as $\frac{1}{k}(A - A_t)$ or $\frac{A}{k} - \frac{A_t}{k}$. It is important that you recognize different ways of writing an expression.

Example 11 : Given $A = kD + A_t$ from the last example, find a formula for A_t, the age of sediments at the surface of a dried-out lake bed.

Solution: Our goal is to get an equation in the form A_t = "something". On the right of the equation there are two terms, kD and A_t. To eliminate a positive term from one side of an equation, you subtract the term from both sides. In this example we subtract kD from both sides and get

$$A_t = A - kD.$$

Activity 11

Assuming this equation is valid, determine the age of the surface sediments if the sedimentation constant was 5000 years per meter and , at a depth of 10 meters, the age was 60,000 years.

In statistics, one equation that you may see is the following called a z-score.

$$Z = \frac{X - \mu}{\sigma} \ .$$

Activity 12

Solve $Z = \frac{X - \mu}{\sigma}$ for X.

Example 12 : Solve P = 2L + 2W, for W.

Solution: Our goal is to get W = "something". To accomplish this we have to eliminate the term 2L and the factor 2. Suppose you decided to eliminate the factor 2 first. Since 2 is a factor, we have to divide **every** term on both sides of the equation by 2. That will give us

$$\frac{P}{2} = \frac{2L}{2} + \frac{2W}{2},$$

which simplifies to

$$\frac{P}{2} = L + W.$$

Now we have to eliminate the term L from the right-hand side, so we subtract it from both sides. Hence,

$$\frac{P}{2} - L = W.$$

Activity 13

How else could you have solved the equation in the last example?

Activity 14

Solve the following equations for the indicated variable.

a) $A = \frac{1}{2}(b_1 + b_2)\, h$, for b_1

b) $V = \frac{1}{3}\, pr^2\, h$, for h

c) $Q = wc(T_1 - T_2)$ for c

d) $Q = \frac{I^2 Rt}{J}$ for R

Section 6: Formulas for Number Sequences

Suppose that at the beginning of 1998 you invested $2,000 at an annual interest rate of 5% compounded annually. Without withdrawing any of the money or making any additional deposits, how much would be in the account at the end of 2004?

To approach this problem, observe that

$$\begin{bmatrix}\text{the amount in the}\\ \text{account at the end}\\ \text{any particular year}\end{bmatrix} = \begin{bmatrix}\text{the amount in the}\\ \text{account at the end}\\ \text{of the previous year}\end{bmatrix} + \begin{bmatrix}\text{the interest earned}\\ \text{on the account}\\ \text{during the year}\end{bmatrix}.$$

The interest earned during the year equals the interest rate 5% = 0.5, times the amount in the account at the end of the previous year. Thus

1) $$\begin{bmatrix}\text{the amount in the}\\ \text{account at the end of}\\ \text{any particular year}\end{bmatrix} = \begin{bmatrix}\text{the amount in the}\\ \text{account at the end}\\ \text{of the previous year}\end{bmatrix} + (.05)\cdot\begin{bmatrix}\text{the amount in the}\\ \text{account at the end}\\ \text{of the previous year}\end{bmatrix}.$$

For each positive integer *n*, let

$$A_n = \begin{bmatrix}\text{the amount in the account}\\ \text{at the end of year } n\end{bmatrix}$$

and let

$$A_0 = \begin{bmatrix}\text{the initial amount}\\ \text{in the account}\end{bmatrix} = \$2000.$$

For any particular year k, substitution into equation 1) above gives

$$\begin{aligned} A_k &= A_{k-1} + .05\cdot A_{k-1} \\ &= (1+.05)\cdot A_{k-1} = 1.05\cdot A_{k-1} \qquad \textbf{\textit{by factoring out } } \boldsymbol{A_{k-1}} \end{aligned}$$

Note: If k is the present year, then k - 1 is the previous year.

The value of your money in the year 2004 can be computed by using the last equation from the previous page and by repeated substitution as follows:

$$A_0 = \$2000$$
$$A_1 = 1.05 \cdot A_0 = 1.05 \cdot \$2000 = \$2100$$
$$A_2 = 1.05 \cdot A_1 = 1.05 \cdot \$2100 = \$2205$$
$$A_3 = 1.05 \cdot A_2 = 1.05 \cdot \$2205 = \$2315.25$$
$$A_4 = 1.05 \cdot A_3 = 1.05 \cdot \$2315.25 = \$2431.01$$
$$A_5 = 1.05 \cdot A_4 = 1.05 \cdot \$2431.01 = \$2552.56$$
$$A_6 = 1.05 \cdot A_5 = 1.05 \cdot \$2552.56 = \$2600.19$$

Thus the amount in the account is \$2600.19 (to the nearest cent) at the end of 2004.

The above calculations were computed using the equation

$$A_k = 1.05 \cdot A_{k-1}$$

from the previous page. In addition, it is important to know how to interpret the subscripts.

A_0 represents the amount in the year 1998

so

A_1 represents the amount in the year 1999.

Recall that using subscripts, the amount of money you had at the end of 1999 could be represented as $\mathbf{A_{1999}}$. Rather than use the years as subscripts we could have just numbered the data values. It is convenient to number the starting value as 0, so $\mathbf{A_0}$ represents the amount you started with in 1998. $\mathbf{A_4}$ is the amount you would have 4 years after 1998 or in the year 2002. In general, $\mathbf{A_n}$ would be the amount you would have n years after 1998.

Example 14 : Suppose a population changes from year to year according to the following equation:

$$p_{n+1} = 2.8(1 - .001p_n)p_n$$

If $p_0 = 10$, find p_1 through p_5.

Solution: $p_0 = 10$

$p_1 = 2.8(1 - .001 \cdot \mathbf{p_0})\mathbf{p_0} = 2.8(1 - .001(10))10 = 27.72$

$p_2 = 2.8(1 - .001(27.72))27.72 = 75.46$

$p_3 = 2.8(1 - .001(75.46))75.46 = 195.3549$

$p_4 = 2.8(1 - .001(195.3549))195.3549 = 440.1358$

$p_5 = 2.8(1 - .001(440.1358))440.1358 = 689.9656$

You may think that the population above keeps increasing, but it hits a bound and settles down to a population of about 642.8.

Activity 15

a) If $a_0 = 2$ and $a_{n+1} = 3a_n + 5$, compute a_1 through a_5.

b) Given the following sequence, find a pattern and describe the pattern with a difference equation.

1, 3, 5, 7, 9, ...

Section 7: Proportions

A proportion is a statement of equality between two ratios, $\frac{a}{b} = \frac{c}{d}$, provided that $b \neq 0$ and $d \neq 0$.

Two ratios are equal when the **cross products** are equal.

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } ad = cb.$$

Let's look at some examples of this.

$$\frac{3}{4} = \frac{9}{12} \text{ since } 3(12) = 4(9).$$

$$\frac{6}{7} \neq \frac{18}{20} \text{ since } 6(20) \neq 7(18).$$

Proportions can be transformed into linear equations, and we can solve them by using methods of linear equations.

Example 15 : Solve the following proportion.

$$\frac{3}{8} = \frac{x}{48}$$

(Read "3 is to 8 as x is to 48")

Solution:

$$\frac{3}{8} = \frac{x}{48}$$

$$48\left(\frac{3}{8}\right) = 48\left(\frac{x}{48}\right)$$

Multiply both sides by 48 to eliminate fractions. Remember you are really multiplying by $\frac{48}{1}$.

$$\frac{48(3)}{8} = x$$

$$6(3) = x$$

$$x = \boxed{18}$$

DO NOT FORGET TO CHECK THE SOLUTION.

You could also solve the last problem by using cross products.

$\frac{3}{8} = \frac{x}{48}$ if and only if 3(48) = 8x

$8x = 3(48)$

$x = \frac{3(48)}{8} = 3 \cdot 6 = 18$

Example 16 : Solve $\frac{t}{7} = \frac{3}{5}$

Solution:

$\frac{t}{7} = \frac{3}{5}$ if and only if 5t = 3(7)

$5t = 21$

$t = \frac{21}{5}$

Activity 16

Solve the following.

a) $\frac{x}{4} = \frac{0.25}{1}$

b) $\frac{x}{6} = \frac{\frac{5}{6}}{\frac{3}{4}}$

Example 17 : On a map 1 inch represents 15 miles. Find the distance represented by 3.5 inches.

Solution: We set up the proportion

$$\frac{1 \text{ in}}{15 \text{ mi}} = \frac{3.5 \text{ in}}{x \text{ mi}}$$

$1x = 15(3.5)$ cross multiply

$x = \boxed{52.5 \text{ miles}}$

Note: It is important to keep your units in the proportion because you can tell if the proportion is set up properly. Notice in our example, inches are in the numerator in both ratios. If you had inches in the numerator in one and miles in the numerator in the other you would know something was wrong.

Activity 17

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a) A 17.3 oz box of bran flakes costs $2.89 and a 12 oz box costs $1.99. How can you tell which is the better buy?

b) In the above problem, if the cost per oz was proportional, how much would you expect to pay for a 17.3 oz box of bran flakes?

Section 8: Percents Using Ratios and Proportions

An important use of ratios is the notion of percent (from the latin *per centum*-per hundred). Thus 70% is the ratio 70/100 and this is quickly reduced to the fraction 7/10 or written as the decimal 0.70. A basketball player is shooting 78% from the foul line. Seventy-eight percent is really a ratio whose denominator is 100, so in this case the ratio of foul shots made to total number of foul shots is 78 to 100. As another example, if a rock specimen is 15% iron by weight, it contains 15 gm of iron in every 100 gm of rock.

If r is any nonnegative real number, then r percent, written r%, is the ratio $\frac{r}{100}$.

Example 18 : Express each percent as a fraction in lowest terms.

a) 40% b) $17\frac{1}{2}$ %

Solution: a) By definition 40% means 40/100. Therefore,

$$40\% = \frac{40}{100} = \frac{2}{5}.$$

b) $$17\frac{1}{2}\ \% = \frac{17\frac{1}{2}}{100} = \frac{\frac{35}{2}}{100} = \frac{35}{200} = \frac{7}{40}.$$

Using proportions we can change fractions to percentages.

Example 19 : Change $\frac{3}{4}$ to a percentage.

Solution: Percent means per 100, so we want to find a fraction equivalent to $\frac{3}{4}$ that has a 100 in the denominator.

$\frac{3}{4} = \frac{x}{100}$ (cross multiply)

$4x = 300$

$x = \frac{300}{4} = 75$

So $\frac{3}{4} = \frac{75}{100} = 75\%$.

Example 20 : Change $\frac{2}{7}$ to a percentage.

Solution: $\frac{2}{7} = \frac{x}{100}$

$7x = 200$

$x = \frac{200}{7} = 28\frac{4}{7}$

Therefore, $\frac{2}{7} = 28\frac{4}{7}\ \%$.

Exercises for Solving Equations, Formulas, and Proportions

1. How many terms do the following expressions have? Name them.

   a) $2x + 3$ b) $2(2x + 7) - 12$ c) $3(2x^2 - 4x + 5) - 2(x - 6)$

2. Simplify the following expressions.

   a) $2(3x + 6)$ b) $5 - 7(4x - 9)$ c) $-5[5(2x - 7) - 3]$

3) Solve the following equations . Check your solutions.

   a) $3x - 7 = 12 - 5x$ b) $5(y-4) +7 = 3y + 10$

   c) $\frac{1}{3}(2a - 8) - 7 = \frac{2}{5}a$ d) $\frac{5x - 6}{9} - \frac{x}{3} = \frac{7}{9}$

   e) $1.2b + 0.03 = 3.2$ f) $\frac{2}{3}(B + 4) = 5 + 3B$

   g) $4(x - 1) - 3(x - 4) = -7$

4. Use your calculator to check if $t = \frac{3}{8}$ is a solution to the equation $\frac{2t}{3} + \frac{1}{2} = \frac{3}{4}$.

5. A formula relating acceleration **a**, velocity **v**, initial velocity $\mathbf{v_0}$, and time **t** is $\mathbf{v = v_0 + at}$. Solve for **t**.
   Hint: Our goal is isolate t on one side of the equation, so we have to eliminate the $\mathbf{v_0}$ *and* ***a*** *from the right side. You have to decide now what you want to eliminate first.*

6. Solve $\mathbf{C_0^2 = C_1^2(1 + 2V)}$ for V.

7. Solve $\mathbf{F = \frac{9}{5} C + 32}$ for C.

8 Solve $\mathbf{Ax + By + C = 0}$ for x.

9 Solve $\mathbf{A = \left(\frac{a + b}{2}\right)h}$ for h.

10. Solve $\mathbf{\Delta L = kL(T - T_0)}$ for T. Note: $\mathbf{\Delta L}$ is treated as one variable.

11. Another equation you will see in a statistics course is $E = Z\frac{\sigma}{\sqrt{n}}$.

Solve this equation for n.

Hint: First get $\sqrt{n}$ by itself on one side of the equation, then square both sides. For example, if you had the equation $\sqrt{x} = 5$, squaring both sides would give you $x = 25$, since $(\sqrt{x})^2 = x$

12. Solve the following proportions.

a) $\frac{y}{7} = \frac{5}{6}$ b) $\frac{x}{3} = \frac{\frac{3}{4}}{\frac{1}{12}}$

13. Using a proportion, change $\frac{7}{8}$ to a percentage.

14. You decide to keep records of your gasoline mileage. Your car is able to go 216 miles on 13.8 gallons of gas. At this rate, how far can the car go on a full tank of 21 gallons?

15. If 7 cans of apple juice cost $2.25, what would 10 cans cost?

16. A recipe for 4 people calls for $1\frac{1}{2}$ tsp. salt. For 7 people, how many tsp. should be used?

SOLVING EQUATIONS, FORMULAS, AND PROPORTIONS

Activity 1:

One possible algebraic expression that has three terms is $3x^2 + 2x - 7$. If we multiply that expression by 4, we get one term: $4(3x^2 + 2x - 7)$.

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Activity 2:

The terms in the expression 6 – 2(3x – 6) are: 6 and –2(3x – 6).
Simplifying, we have

$$\begin{aligned} & 6 - 2(3x - 6) \\ &= 6 - 6x + 12 \quad \textit{distribute the } -2 \textit{ to } (3x-6). \\ &= -6x + 18 \quad \textit{combine } 6 \textit{ and } 12 \end{aligned}$$

****
Activity 3:

$$\begin{aligned} & 4\{3 + 5(x - 6)\} \\ &= 12 + 20(x - 6) \quad \textit{distribute the } 4 \textit{ to } 3 \textit{ and } 5 \\ &= 12 + 20x - 120 \quad \textit{distribute the } 20 \textit{ to } (x-6) \\ &= 20x - 108 \quad \textit{combine } 12 \textit{ and } -120 \end{aligned}$$

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Activity 4:

Three equations that are equivalent to $4x + 3 = 10 - 3x$ *are*:

1) $4x = 7 - 3x$
2) $7x = 7$
3) $x = 1$

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Activity 5:

$a)$ *If* $20x = \frac{1}{3}$, *then* $60x = 1$ (*multiply both sides by* 3)

$b)$ *If* $8y = 0.025$, *then* $800y = 2.5$ (*multiply both sides by* 100)

$c)$ *If* $7a = 10$, *then* $8a = 10 + 1a$ (*add* 1*a to both sides*)

Activity 6:

$a)$ $6(b+2)+12=30$

$6b-12+12=30$	*distribute 6 to (b –2)*
$6b=30$	*add –12 and 12*
$b=5$	*divide both sides of the equation by 6*

$b)$ $8+4(5x-1)=10$

$8+20x-4=10$	*distribute the 4 to (5x – 1)*
$20x+4=10$	*subtract 8 and 4*
$20x=6$	*subtract 4 from both sides of the equation*
$x=\frac{6}{20}=\frac{3}{10}$	*divide both sides of the equation by 20, then simplify*

$c)$ $6-2(3x-2)=7$

$6-6x+4=7$	*distribute the –2 to (3x – 2)*
$10-6x=7$	*add 6 and 4*
$-6x=-3$	*subtract 10 from both sides of the equation*
$x=\frac{-3}{-6}=\frac{3}{6}=\frac{1}{2}$	*divide both sides of the equation by –6, then simplify*

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Activity 7:

One way:

$3(4x-8)-2(3x+5)=4(x+7)$

$12x-24-6x-10=4x+28$	*distribute the 3, -2 and 4*
$6x-34=4x+28$	*combine like terms on the left*
$6x=4x+62$	*add 34 to both sides of the equation*
$2x=62$	*subtract 4x from both sides of the equation*
$x=31$	*divide both sides of the equation by 2*

Another way:

$3(4x-8)-2(3x+5)=4(x+7)$

$3(4x-8)=2(3x+5)+4(x+7)$	*add 2(3x + 5) to both sides of the equation*
$12x-24=6x+10+4x+28$	*distribute the 3, 2 and 4*
$12x-24=10x+38$	*add 6x and4x ; 10 and 28*
$12x=10x+62$	*add 24 to both sides of the equation*
$2x=62$	*subtract 10x from both sides of the equation*
$x=31$	*divide both sides of the equation by 2*

Activity 8: $\frac{x+5}{3}+6=\frac{x}{2}$

$6\left(\frac{x+5}{3}\right)+6(6)=6\left(\frac{x}{2}\right)$ *multiply all terms on both sides of the equation by 6*

$2(x+5)+36=3x$ *simplify*

$2x+10+36=3x$ *distribute the 2 to (x + 5)*

$2x+46=3x$ *add 10 and 36*

$46=x$ *subtract 2x from both sides of the equation*

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Activity 9: $\frac{1}{7}(x+8)=\frac{2}{5}(x-4)$

$\frac{1}{7}\left(\frac{32}{3}+8\right)=\frac{2}{5}\left(\frac{32}{3}-4\right)$ *substitute $\frac{32}{3}$ for x into the given equation*

$\frac{1}{7}\left(\frac{32}{3}+\frac{24}{3}\right)=\frac{2}{5}\left(\frac{32}{3}-\frac{12}{3}\right)$ *get a common denominator of 3*

$\frac{1}{7}\left(\frac{56}{3}\right)=\frac{2}{5}\left(\frac{20}{3}\right)$ *combine fractions within each set of parentheses*

$\frac{8}{3}=\frac{8}{3}$ *multiply the fractions on each side of the equation*

Since $\frac{8}{3}=\frac{8}{3}$, then $\frac{32}{3}$ is a solution to the equation.

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Activity 10:

1) $Age = k \bullet Depth$

$k=\frac{Age}{Depth}$ *divide both sides by Depth*

2) $k=\frac{4500}{5}=900$

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Activity 11:

$A_t = A-kD$

$A_t = 60{,}000 - 5000(10)$

$= 10{,}000\ years$

Activity 12:

$$Solve \quad Z = \frac{X - \mu}{\sigma} \; for \; X.$$

$\sigma Z = X - \mu$ *multiply both sides of the equation by* σ

$\sigma Z + \mu = X$ *add* μ *to both sides of the equation*

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Activity 13:

$P = 2L + 2W$

$P - 2L = 2W$ *subtract* $2L$ *from both sides*

$\frac{P - 2L}{2} = W$ *divide both sides by* 2

$\frac{P}{2} - \frac{2L}{2} = W$ *separate fraction into 2 fractions*

$\frac{P}{2} - L = W$ *divide 2 by 2 to get* L

****

Activity 14:

$a)$ $A = \frac{1}{2}(b_1 + b_2)h, \; for \; b_1$

$2A = (b_1 + b_2)h$ *multiply both sides by* 2

$\frac{2A}{h} = b_1 + b_2$ *divide both sides by* h

$\frac{2A}{h} - b_2 = b_1$ *subtract* b_2 *from both sides*

$b)$ $V = \frac{1}{3}pr^2h, \; for \; h$

$3V = pr^2h$ *multipy both sides by* 3

$\frac{3V}{pr^2} = h$ *divide both sides by* pr^2

$c)$ $Q = wc(T_1 - T_2) \; for \; c$

$\frac{Q}{w(T_1 - T_2)} = c$ *divide both sides by* $w(T_1 - T_2)$

$d)$ $Q = \frac{I^2 Rt}{J} \; for \; R$

$QJ = I^2 Rt$ *multiply both sides by* J

$\frac{QJ}{I^2 t} = R$ *divide both sides by* $I^2 t$

Activity 15:

a) If $a_{n+1} = 3a_n + 5$ *and* $a_0 = 2$, *then*

$$n = 0: \quad a_1 = 3a_0 + 5 = 3(2) + 5 = 11$$
$$n = 1: \quad a_2 = 3a_1 + 5 = 3(11) + 5 = 38$$
$$n = 2: \quad a_3 = 3a_2 + 5 = 3(38) + 5 = 119$$
$$n = 3: \quad a_4 = 3a_3 + 5 = 3(119) + 5 = 362$$
$$n = 4: \quad a_5 = 3a_4 + 5 = 3(362) + 5 = 1091$$

b) $a_{n+1} = a_n + 2 \qquad a_0 = 1\,;\, a_1 = 3\,;\, a_2 = 5\,;\, a_3 = 7\,;\, a_4 = 9$

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Activity 16:

a)
$$\frac{x}{4} = \frac{0.25}{1}$$
$$1x = 4(0.25)$$
$$x = 1$$

b)
$$\frac{x}{6} = \frac{\frac{5}{6}}{\frac{3}{4}}$$

$$\frac{3}{4}x = 6\left(\frac{5}{6}\right) \qquad \textit{cross multiply}$$

$$\frac{3}{4}x = 5 \qquad \textit{multiply } 6 \textit{ and } \frac{5}{6}$$

$$\frac{4}{3} \bullet \frac{3}{4}x = 5 \bullet \frac{4}{3} \qquad \textit{multiply each side by the reciprocal of } \frac{3}{4}$$

$$x = \frac{20}{3} = 6\frac{2}{3} \qquad \textit{multiply } 5 \textit{ and } \frac{4}{3}$$

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Activity 17:

a) $\dfrac{\$2.89}{17.3\ oz} \approx \$0.167\ \textit{per ounce} \qquad \dfrac{\$1.99}{12\ oz} \approx \$0.166\ \textit{per ounce}$

Therefore, the 12 ounce box is a better buy.