Math 171 Proficiency Packet on Fractions

Section 1: Introduction

Recall that the set of whole numbers, W = {0, 1, 2, 3, 4,...}, was extended to the set of integers I = {...,-2, -1, 0, 1, 2, ...} . In this unit we extend the set of integers to a number system called the **rational numbers**. Rational numbers were developed because there are instances when whole numbers and integers cannot fully describe or quantify the situation. For example, how would you describe the portion of the large rectangle shaded in the following diagram?

xxxxxxxxxxxxx	xxxxxxxxxxxxx	
xxxxxxxxxxxxx		

You can see that we have to extend the integers in order to handle a situation like this.

Definition: A **fraction** is a number of the form $\frac{a}{b}$, where a and b are integers and b $\neq$ 0.

Some examples of fractions are:

$\frac{2}{3}$	$\frac{3}{5}$	$\frac{6}{7}$	$\frac{8}{5}$
Two-thirds	Three-fifths	Six-sevenths	Eight-fifths

Definition: For the fraction $\frac{a}{b}$, a and b are called the **terms** of the fraction. More specifically, a is called the **numerator** and b is called the **denominator.**

Example 1: The terms of the fraction $\frac{2}{3}$ are 2 and 3. The 2 is called the numerator, and the 3 is called the denominator.

Example 2: An integer such as 5 may also be put in fraction form, since 5 can be written as $\frac{5}{1}$. In this case, 5 is the numerator, and 1 is the denominator.

Definition: A **proper fraction** is a fraction in which the numerator is less than the denominator. If the numerator is greater than or equal to the denominator, the fraction is called an **improper fraction.**

In example 1, $\frac{2}{3}$ is a **proper** fraction and in example 2, $\frac{5}{1}$ is an **improper** fraction.

Section 2: Fraction Models

There are three categories of models used in developing fraction concepts: area models, set models, and length or measurement models. We will try to use all three models in our development of fractions.

When using the three models for fractions,

1) we have to identify what the "whole" or "1" is,
2) we divide the "whole" into a number (say y) of **equal** (in some sense) parts,
3) we state the number (say x) of those equal parts being considered.

The symbol, $\frac{x}{y}$, is assigned to the result of this process. For example, in the figure below, we took the "whole", which is the larger outer rectangle, and divided it up into 8 **equal** parts and we shaded (considered) 5 of these parts. The result is that we have 5 eighths of the whole, which is written symbolically as $\frac{5}{8}$. The following is an example of an **area model**, representing $\frac{5}{8}$.

xxxxxxxxxxxxx	xxxxxxxxxxxxxxx		
xxxxxxxxxxxxx	xxxxxxxxxxxxxxx	xxxxxxxxxxxxxxx	

Example 1: In the figure below, why is it incorrect to say that $\frac{1}{3}$ of the rectangle is shaded?

	xxxxxxxxxxx xxxxxxxxxxx	

Solution: We took the large rectangle and divided it into three regions or parts, one of the three regions is shaded. The problem is that the regions are not equal. When we look at a fraction like $\frac{1}{3}$, we are assuming that the "whole" has been divided into three equal parts.

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Now You Try (Section 2.1)

1) Divide the rectangle into 3 congruent parts and shade 2 of the parts.

2) Divide the rectangle into 4 congruent parts and shade 3 of the parts.

3) Divide the rectangle into 6 congruent parts and shade 4 of the parts.

4) What fractions do each of the shaded regions represent in parts a - c?

1) _____ 2) _____ 3) _____

(Answers to **<u>Now You Try</u>** (Section 2.1) *are found on page 28.)*

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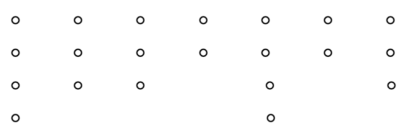
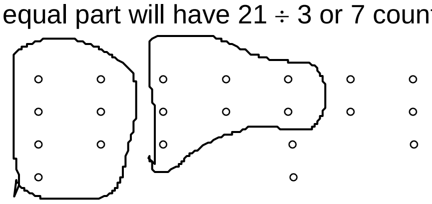
The following example illustrates the **set model** for fractions.

Example 1: Divide the set of counters below into 3 equal parts and take 2 of these subsets. In this case the "whole" is represented by the complete set of counters. What fraction of the whole is represented?



Solution: Since there are a total of 21 counters and we want to divide them up into 3 equal parts each equal part will have 21 ÷ 3 or 7 counters.



The fraction represented is $\frac{2}{3}$.

<u>Now You Try</u> (Section 2.2)

1) Divide the set of counters below into 4 matching subsets and take 2 of these subsets.

◦ ◦ ◦ ◦
◦ ◦ ◦ ◦
◦ ◦ ◦ ◦

2) Divide the set of counters below into 5 matching subsets and take 3 of these subsets.

◦ ◦ ◦ ◦
◦ ◦ ◦ ◦
◦ ◦

3) Divide the set of counters below into 7 matching subsets and take 2 of these subsets.

◦ ◦ ◦ ◦
◦ ◦ ◦ ◦
◦ ◦ ◦ ◦
◦ ◦ ◦ ◦
◦ ◦ ◦
◦ ◦

4) What fractions are represented in parts a - c?

1) _____ 2) _____ 3) _____

(Answers to **<u>Now You Try</u>** (Section 2.2) *are found on page 28.)*

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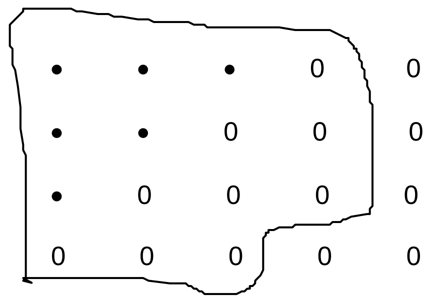
Example 2: The shaded dots have a value of $\frac{2}{5}$. Draw a circle around the "whole" or "1".

• • • 0 0
• • 0 0 0
• 0 0 0 0
0 0 0 0 0

Solution: Since we have 6 shaded dots and the 2 in the numerator tells us that we have 2 equal subsets, each subset would have to have 3 shaded dots in it. The 5 in the denominator tells us that there are 5 subsets, this means that the "whole" would be represented by 15 dots (5 subsets with 3 dots each).



Another method to solve this problem would be to use a **proportion**.

1) Count how many shaded dots are given. In this case, there are 6.

2) Make a proportion with the original fraction on the left and $\frac{6}{x}$ on the right.

$$\frac{2\ shaded}{5\ total} = \frac{6\ shaded}{x\ total}$$

3) Solve the proportion by first cross multiplying.

$$2x = 5(6)$$
$$2x = 30$$

4) Then, divide both sides of the equation by 2.

$$\frac{2x}{2} = \frac{30}{2}$$

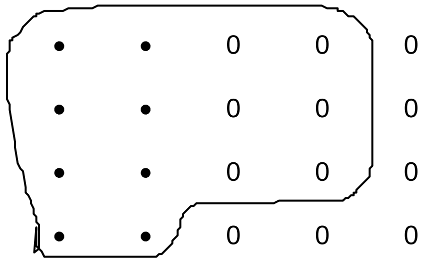
$$x = 15$$

Therefore, there are 15 total dots that need to be circled.

In a previous course, you may recall a question such as, "$\frac{2}{5}$ of what number is 6?". Example 2 above is really asking you the same thing, the "whole" we are looking for in the example is the "number" you were looking for in the question.

Example 3: $\frac{4}{7}$ of what number is 8?

Solution: This question is asking you to find the "whole" if the 8 shaded dots represent $\frac{4}{7}$.



The 8 shaded dots have to be divided into 4 equal subsets, so each subset will have 2 dots. In the "whole" there are 7 subsets, so the "whole" will have 14 dots. Hence, $\frac{4}{7}$ of **14** is 8.

We could also use a proportion to solve:

$$\frac{4\ shaded}{7\ total} = \frac{8\ shaded}{x\ total}$$
$$4x = 56$$
$$x = 14$$

Therefore, $\frac{4}{7}$ of **14** is 8.

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<u>Now You Try</u> (Section 2.3)

1) The shaded dots represent $\frac{5}{7}$. Draw a circle around the "whole" or "1".

•	•	•	0	0
•	•	0	0	0
0	0	0	0	0
0	0	0	0	0

2) The shaded dots represent $\frac{3}{5}$. Draw a circle around the "whole" or "1".

```
•   •   •   •   0   0
•   •   •   •   0   0
•   •   0   0   0   0
•   •   0   0   0   0
0   0   0   0   0   0
0   0   0   0   0   0
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(Answers to **<u>Now You Try</u>** (Section 2.3) *are found on page 29.)*

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The following example illustrates the **length** or **measurement model.**

Example 1: If we take a line segment (the whole or one) and divide it up into 8 equal parts and then we consider 3 of those equal parts, the result is that we have 3 eighths, which is written symbolically as $\frac{3}{8}$.

←—|—|—|—|—|—|—|—|—|⟶

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8} = 1$

Example 2: Given the following number line, state what fraction A represents.

←—|—|—|—|—|—|—|—|⟶

0 A 1

Solution: Since there are 7 equal parts from 0 to 1, each tic mark represents $\frac{1}{7}$. So A = $\frac{4}{7}$.

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<u>Now You Try</u> (Section 2.4)

1) Given the following number line, what fractions are represented by the letters A, B, C, and D?

|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|—|

0 A B 1 C 2 D 3 4

A ______ B ______ C ______ D ______

2) Given the following number line, what fractions are represented by the letters A and B?

|—|—|—|—|—|—|—|—|—|
0 A 1 B 2

A _____ B _____

3) Approximate fractions represented by the letters A and B.

|————————|————————|
0 A 1 B 2

A _____ B _____

(Answers to **Now You Try** (Section 2.4) *are found on page 29.)*

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Section 3: Equivalent Fractions

Definition:	When two fractions represent the same amount with respect to the same "whole", the two fractions are said to be **equivalent**.

Example 1: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \ldots$ All of these fractions are equivalent because they represent the same value on the number line.

←—|————————|————————|—→
0 $\frac{1}{2}$ 1

←—|————|————|————|————|—→
0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

←—|———|———|———|———|———|———|——→
0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1

If a, b, and c are integers and b and c are not 0, then it is always true that

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a}{b} = \frac{a \div c}{b \div c}$$

in other words, if the numerator and the denominator of a fraction are **multiplied** or divided by the same nonzero number, the resulting fraction is **equivalent** to the original fraction.

Example 2: Write $\frac{2}{5}$ as an equivalent fraction with a denominator of 25.

Solution: The denominator of the original fraction is 5. The fraction we are trying to find must have a denominator of 25. We know that if we multiply 5 by 5, we get 25. The property above indicates that we can multiply the denominator by 5 as long as we do the same to the numerator.

$$\frac{2}{5} = \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

The fraction $\frac{2}{5}$ is equivalent to $\frac{10}{25}$.

Example 3: Write $\frac{12}{18}$ as an equivalent fraction with a denominator of 6.

Solution: If we divide the original denominator 18 by 3, we get 6. The property above indicates that if we divide both the numerator and denominator by 3, we will get an equivalent fraction.

$$\frac{12}{18} = \frac{12 \div 3}{18 \div 3} = \frac{4}{6}$$

The fraction $\frac{12}{18}$ is equivalent to $\frac{4}{6}$.

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Now You Try (Section 3.1)

1) Write $\frac{7}{8}$ as an equivalent fraction with a denominator of 48.

2) Write $\frac{24}{40}$ as an equivalent fraction with a denominator of 5.

(Answers to **Now You Try** (Section 3.1) *are found on page 29.)*

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if $ad = bc$. In other words, 2 fractions are equal when their cross products are equal.

Example 1: Are $\frac{4}{5}$ and $\frac{11}{15}$ equivalent?

Solution: Two fractions are equal if their cross products are equal. Since $4 \cdot 15 \neq 5 \cdot 11$, $\frac{4}{5}$ and $\frac{11}{15}$ are **not** equivalent.

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<u>Now You Try</u> (Section 3.2)

Determine if the following pairs of fractions are equivalent.

1) $\frac{7}{10}, \frac{9}{14}$ ____________________________________

2) $\frac{3}{4}, \frac{24}{32}$ ____________________________________

(Answers to **<u>Now You Try</u>** (Section 3.2) *are found on page 29.)*

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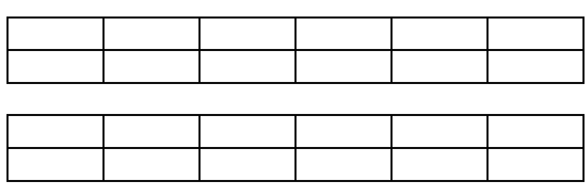
<u>Section 4:</u> Comparing Fractions

Now that we can determine if two fractions are equivalent, the next topic is to determine which is <u>greater</u>.

Models are useful in determining which fraction is larger, so before we find a general formula, try the following examples:

Example 1: Shade the regions that correspond to the fractions $\frac{2}{3}$ and $\frac{7}{12}$ on the rectangles below.



Which fraction is larger? __________

Solution:

1) Shade $\frac{2}{3}$ by dividing the region up into 3 sections (4 rectangles in each section), then shade 2 of the sections.

xxxx	xxxx	xxxx	xxxx		
xxxx	xxxx	xxxx	xxxx		

2) Shade $\frac{7}{12}$ by shading in 7 of the 12 equal parts.

xxxx	xxxx	xxxx	xxxx		
xxxx	xxxx	xxxx			

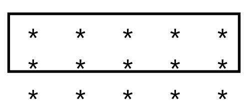
Therefore, $\frac{2}{3}$ is larger since there are 8 of the 12 equal parts shaded.

Example 2: Use the sets below to determine which is the larger fraction:

$\frac{2}{3}$ or $\frac{3}{5}$

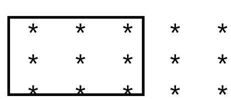
*	*	*	*	*		*	*	*	*	*
*	*	*	*	*		*	*	*	*	*
*	*	*	*	*		*	*	*	*	*

Solution: 1) Circle $\frac{2}{3}$ of the stars in the 1st grouping.



10 stars circled

2) Circle $\frac{3}{5}$ of the stars in the 2nd grouping.



9 stars circled

Therefore, $\frac{2}{3}$ is larger since there were 10 stars circled.

Notice that in the 2 model examples, the visual representations were actually showing the common denominators for the fractions. In example 1, there were 12 equal parts and 12 is the common denominator between 3 and 12. In example 2, there were 15 stars and 15 is the common denominator between 3 and 5.

General Rule for Determining Which Fraction is Greater

Case 1: **Fractions with common denominators.**

$\frac{a}{n} < \frac{b}{n}$ if and only if $a < b$.

In other words, if the denominators are the same, compare the numerators.

Case 2: **Fractions with like denominators.**

Now consider $\frac{a}{b}$ and $\frac{c}{d}$. We know $\frac{a \cdot d}{b \cdot d} = \frac{c \cdot b}{d \cdot b}$ so we can than state that

$\frac{a}{b} < \frac{c}{d}$ if and only if $a \cdot d < c \cdot b$.

In other words, if the denominators are different, compare their cross products.

Example 1: Which fraction is greater: $\frac{3}{8}$ or $\frac{7}{17}$?

Solution: Let's start by finding equivalent fractions for both so that they have the same denominator. Multiply the top and the bottom of $\frac{3}{8}$ by 17 and multiply the top and bottom of $\frac{7}{17}$ by 8. This gives us

$$\frac{3}{8} = \frac{3 \cdot 17}{8 \cdot 17} = \frac{51}{136} \text{ and } \frac{7}{17} = \frac{7 \cdot 8}{17 \cdot 8} = \frac{56}{136}.$$

You can see that $\frac{7}{17}$ is greater than $\frac{3}{8}$ since 56 is greater than 51.

The shortcut method is given in case 2 above.

$\frac{3}{8} < \frac{7}{17}$ because $3 \cdot 17 < 7 \cdot 8$.

Example 2: Which fraction is greater: $\frac{2}{15}$ or $\frac{7}{53}$?

Solution: Since $2(53) > 7(15)$, $\frac{2}{15}$ is greater than $\frac{7}{53}$.

Now You Try (Section 4.1)

State which fraction is greater.

1) $\frac{2}{5}$ or $\frac{4}{7}$

2) $\frac{13}{20}$ or $\frac{9}{15}$

(Answers to **Now You Try** (Section 4.1) *are found on page 29.)*

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Fractions have an interesting property that whole numbers and integers do not have. If you take two whole numbers such as 5 and 6, there is not a whole number between them, but if you are given any two fractions you can always find another fraction between them. This property is called the **density property**.

Example 1: Find at least one rational number between $\frac{5}{6}$ and $\frac{7}{8}$.

Solution: To find a rational number between $\frac{5}{6}$ and $\frac{7}{8}$, you need to find equivalent fractions for each so that they have common denominators: $\frac{5\cdot 4}{6\cdot 4}=\frac{20}{24}$ and $\frac{7\cdot 3}{8\cdot 3}=\frac{21}{24}$.

However, we cannot find a whole number numerator between 20 and 21, so we need to use a larger common denominator, such as 48: $\frac{5\cdot 8}{6\cdot 8}=\frac{40}{48}$ and $\frac{7\cdot 6}{8\cdot 6}=\frac{42}{48}$.

Since $\frac{41}{48}$ is between $\frac{40}{48}$ and $\frac{42}{48}$, $\boxed{\frac{41}{48} \text{ is between } \frac{5}{6} \text{ and } \frac{7}{8}}$.

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Now You Try (Section 4.2)

1) Find the fraction between $\frac{1}{2}$ and $\frac{5}{7}$.

2) Find the fraction between $\frac{4}{5}$ and $\frac{7}{10}$.

(Answers to **Now You Try** (Section 4.2) *are found on page 29.)*

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Section 5: Simplifying Fractions to Lowest Terms

By looking at the following examples,

$$\frac{2}{3}=\frac{2\cdot 5}{3\cdot 5}=\frac{10}{15} \qquad \frac{6}{7}=\frac{6\cdot 9}{7\cdot 9}=\frac{54}{63}$$

can you think of a way to start with $\frac{54}{63}$ and simplify it to get $\frac{6}{7}$?

To simplify a fraction to lowest terms, we have to divide the numerator and the denominator by all the **factors** they have in common. Recall that factors are numbers that are multiplied together. When we divide the numerator and the denominator of a fraction by the same nonzero number, we get an equivalent fraction. So to simplify a fraction, all we have to do is recognize what factors the numerator and the denominator have in common and then divide the numerator and the denominator by the common factors. Let's do some examples of simplifying fractions.

Definition:	A fraction is said to be in **lowest terms** if the numerator and the denominator have no factors in common other than the number 1.

Example 1: Write $\frac{12}{15}$ in the lowest terms.

Solution: We need to find the factors of 12 and 15 so that we can find one they have in common.

Factors of 12 are: 1, 2, 3, 4, 6, 12

Factors of 15 are: 1, 3, 5, 15

Since 3 is the only factor they have in common, we will divide the numerator and denominator by 3.

$$\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

In lowest terms, since 4 and 5 have no other factor besides 1 in common.

Example 2: Write $\frac{16}{24}$ in lowest terms.

Solution: Factors of 16 are: 1, 2, 4, 8, 16

Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

Since 16 and 24 have 2, 4, and 8 in common, we will choose the highest factor, 8.

$$\frac{16 \div 8}{24 \div 8} = \frac{2}{3}.$$

In lowest terms, since 2 and 3 have no other factor besides 1 in common.

Notice that you would also write $\frac{16}{24}$ *in lowest terms, by simply dividing numerator and denominator by 2 until they have no factor in common but 1.*

$$\frac{16 \div 2}{24 \div 2} = \frac{8 \div 2}{12 \div 2} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}.$$

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Now You Try (Section 5)

Write the following fractions in lowest terms.

1) $\frac{54}{72}$

2) $\frac{16}{28}$

(Answers to **Now You Try** (Section 5) *are found on page 29.)*

<u>Section 6:</u> Adding and Subtracting Fractions

<u>Case 1:</u> Fractions with like denominators.

(1) Add or subtract the numerator and place the answer over the common denominator.

(2) Write the answer in lowest terms.

Example1:

a) Add: $\frac{3}{8}+\frac{1}{8}$.

b) Subtract: $\frac{7}{10}-\frac{3}{10}$.

Solution:

a) Since the denominators are the same, add the numerators and keep the denominator the same.

$$\frac{3}{8}+\frac{1}{8} = \frac{4}{8}$$

Since $\frac{4}{8}$ is not in lowest terms, divide numerator and denominator by 4 to get $\frac{1}{2}$.

b) Again the denominators are the same, so we just subtract the numerators and place that answer over the common denominator.

$$\frac{7}{10}-\frac{3}{10} = \frac{4}{10}$$

Since $\frac{4}{10}$ is not in lowest terms, divide numerator and denominator by 2 to get $\frac{2}{5}$.

<u>Case 2:</u> Fractions with unlike denominators.

(1) Find equivalent fractions for each original fraction, so that both fractions have a common denominator.

(2) Follow the procedure for like denominators.

Example 1:

Add: $\frac{5}{12}+\frac{1}{18}$.

Solution:

Since we have unlike denominators, we need to determine a common denominator between 12 and 18. To do this, we need to look at the multiples of 12 and 18 and find the smallest common multiple.

Multiples of 12 are: 12, 24, 36, 48, …

Multiples of 18 are: 18, 36, 54, 72, …

Since 36 is the 1st multiple in common, that will be our common denominator.

(You could also just multiply their denominators together to find a common multiple but this often leads to large numbers which are not easily simplified.)

Next , we need to write equivalent fractions for each of our original fractions, having a common denominator of 36.

$\frac{5\cdot 3}{12\cdot 3}=\frac{15}{36}$ *Since 12 · 3 = 36, we multiply numerator and denominator by 3.*

$\frac{1\cdot 2}{18\cdot 2}=\frac{2}{36}$ *Since 18 · 2 = 36, we multiply numerator and denominator by 2.*

Now that we have common denominators, we can add the numerators and place that over the common denominator,

$$\frac{15+2}{36}=\boxed{\frac{17}{36}}$$

Example 2: Subtract: $\frac{4}{15}-\frac{2}{9}$.

Solution: Multiples of 15 are: 15, 30, 45, 60,…

Multiples of 9 are: 9, 18, 27, 36, 45,…

Since 45 is the 1st multiple in common, that will be our common denominator.

$\frac{4\cdot 3}{15\cdot 3}=\frac{12}{45}$ *Multiply numerator and denominator by 3.*

\- $\frac{2\cdot 5}{9\cdot 5}=\frac{10}{45}$ *Multiply numerator and denominator by 5.*

$\boxed{\frac{2}{45}}$ *Subtract numerators and place answer over common denominator.*

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<u>Now You Try</u> (Section 6)

1) Add: $\frac{3}{8}+\frac{2}{5}$ 3) Subtract: $\frac{7}{15}-\frac{3}{10}$

2) Add: $\frac{7}{9}+\frac{1}{6}$ 4) Subtract: $\frac{2}{3}-\frac{3}{8}$

(Answers to **Now You Try** (Section 6) *are found on page 30.)*

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<u>Section 7:</u> Multiplying Fractions

<u>To Multiply Fractions</u>

(1) Divide out any common factors between any numerator and any denominator.

(2) Multiply remaining numbers in the numerator, then multiply remaining numbers in the denominator.

(3) Place the product of the numerators over the product of the denominators.

(4) Check that the answer is in lowest terms.

Example 1: Multiply: $\frac{3}{5}\cdot\frac{2}{7}$.

Solution: Since there are no common factors between any of the numerators and denominators, we simply multiply the numerators, then multiply denominators.

$$\frac{3}{5}\cdot\frac{2}{7} = \frac{3\cdot 2}{5\cdot 7} = \boxed{\frac{6}{35}}$$ which is in lowest terms.

Example 2: Multiply: $\frac{25}{32}\cdot\frac{8}{20}$.

Solution:

$$\frac{{}^{5}\cancel{25}}{32}\cdot\frac{8}{\cancel{20}_{4}}$$ *Divide 20 and 25 by their common factor of 5.*

$$\frac{{}^{5}\cancel{25}}{\cancel{32}_{4}}\cdot\frac{\cancel{8}^{1}}{\cancel{20}_{4}}$$ *Divide 8 and 32 by their common factor of 8.*

$$\frac{5\cdot 1}{4\cdot 4} = \boxed{\frac{5}{16}}$$ *Multiply remaining numbers in the numerator and denominator.*

Answer is in lowest terms.

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<u>Now You Try</u> (Section 7)

Multiply:

1) $\frac{8}{5}\cdot\frac{25}{24}$

3) $\frac{9}{20}\cdot\frac{8}{15}$

2) $\frac{5}{18}\cdot\frac{8}{35}$

4) $\frac{12}{25}\cdot\frac{5}{6}$

(Answers to **<u>Now You Try</u>** (Section 7) *are found on page 30.)*

Section 8: Dividing Fractions

Definition: If $\frac{a}{b}$ is a fraction, the fraction $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

To Divide Fractions

(1) Rewrite the 1st fraction as it is given.

(2) Change the division sign to a multiplication sign.

(3) Write the reciprocal of the 2nd fraction.

(4) Use the rules for multiplying fractions.

Example 1: Divide: $\frac{1}{5} \div \frac{2}{3}$.

Solution: $\frac{1}{5} \cdot \frac{3}{2}$ *Multiply by the reciprocal.*

$\frac{1 \cdot 3}{5 \cdot 2} = \boxed{\frac{3}{10}}$ *Multiply numerators, then multiply denominators. Answer in lowest terms.*

Example 2: Divide: $\frac{13}{28} \div \frac{26}{35}$.

Solution: $\frac{13}{28} \cdot \frac{35}{26}$ *Multiply by the reciprocal.*

$\frac{{}^{1}\cancel{13}}{\cancel{28}_4} \cdot \frac{\cancel{35}^{5}}{\cancel{26}_2}$ *Divide out any common factors between any numerator and any denominator (divide 28 and 35 by 7 ; 13 and 26 by 13).*

$\frac{1 \cdot 5}{4 \cdot 2} = \boxed{\frac{5}{8}}$ *Multiply numerators, then multiply denominators. Answer is in lowest terms.*

Example 3: Divide: $\frac{3}{2} \div 9$.

Solution: $\frac{3}{2} \div \frac{9}{1}$ *Write 9 as a fraction by placing a 1 in the denominator.*

$\frac{3}{2} \cdot \frac{1}{9}$ *Multiply by the reciprocal.*

$$\frac{\not{3}^{1}}{2} \cdot \frac{1}{\not{9}_{3}}$$ *Divide 3 and 9 by 3.*

$$\frac{1 \cdot 1}{2 \cdot 3} = \boxed{\frac{1}{6}}$$ *Multiply numerators, then multiply denominators. Answer is in lowest terms.*

****

Now You Try (Section 8)

Divide:

1) $\frac{3}{8} \div \frac{15}{4}$ 2) $\frac{24}{25} \div \frac{6}{5}$ 3) $\frac{4}{9} \div 6$

(Answers to **Now You Try** (Section 8) *are found on page 30.)*

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Section 9: Mixed Numbers

A number of the form $3\frac{2}{5}$ is called a mixed number because it is composed of a whole number and a fraction. Sometimes people misinterpret it to mean 3 times $\frac{2}{5}$. In general, a number of the form $A\frac{b}{c}$ means $A + \frac{b}{c}$, so $3\frac{2}{5}$ = $3 + \frac{2}{5}$. Mixed numbers are helpful to get a better idea of the size of a number. For example, when you look at an improper fraction such as $\frac{341}{17}$, it is difficult to get a feeling for its size. However, if we write $\frac{341}{17}$ as the mixed number, $21\frac{4}{17}$, we have a better feeling for its size.

To Change from an Improper Fraction to a Mixed Number

(1) Divide the denominator into the numerator. This quotient becomes the whole number part of the mixed number.

(2) If the denominator does not divide evenly into the numerator, place the remainder over the denominator. This becomes the fractional part of the mixed number.

Example 1: Change $\frac{37}{5}$ to a mixed number.

Solution:

$$\begin{array}{r} 7 \\ 5\overline{)37} \\ \underline{35} \\ 2 \end{array}$$

Divide the denominator into the numerator.

Since 2 is the remainder, place that over the denominator 5.
Write as a mixed number.

Therefore, $\frac{37}{5}$ = $\boxed{7\frac{2}{5}}$.

To Change from a Mixed Number to an Improper Fraction

(1) Multiply the whole number and the denominator, add the result to the numerator.

(2) Place the answer from step (1) over the denominator.

Example 1: Change $5\frac{4}{9}$ to an improper fraction.

Solution: $5\frac{4}{9} = \frac{5 \cdot 9 + 4}{9} = \frac{45+4}{9} = \frac{49}{9}$

Therefore, $5\frac{4}{9} = \boxed{\frac{49}{9}}$.

****

Now You Try (Section 9.1)

1) Change the following improper fractions to mixed numbers.

a) $\frac{41}{9}$ b) $\frac{527}{21}$

2) Change the following mixed numbers to improper fractions.

a) $8\frac{3}{7}$ b) $12\frac{5}{9}$

(Answers to **Now You Try** (Section 9.1) *are found on page 30.)*

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Adding and Subtracting Mixed Numbers

Case 1: (1) Leave the mixed numbers as mixed numbers.

(2) Find a common denominator and write equivalent fractions for the fractional parts.

(3) Add or subtract the fractional parts. When subtracting, you may need to borrow. *(See example below.)*

(4) Add or subtract the whole number parts.

(5) Write the fractional part in lowest terms.

Example 1: Add: $3\frac{2}{5} + 2\frac{2}{3}$.

Solution:

$3\frac{2}{5} = 3\frac{6}{15}$ *Find a common denominator of 15.*

$+\ 2\frac{2}{3} = +2\frac{10}{15}$ *Write equivalent fractions.*

$5\frac{16}{15}$ *Add the whole numbers, then add the fractions.*

Since $\frac{16}{15}$ is improper, we need to change it to a mixed number $\left(\frac{16}{15} = 1\frac{1}{15}\right)$.

Now we can add $1\frac{1}{15}$ to 5, giving us $6\frac{1}{15}$.

Therefore, $3\frac{2}{5} + 2\frac{2}{3} = \boxed{6\frac{1}{15}}$.

Example 2: Subtract: $4\frac{1}{6} - 2\frac{1}{4}$.

Solution:

$$4\frac{1}{6} = 4\frac{2}{12}$$ *Find a common denominator of 12.*

$$-2\frac{1}{4} = -2\frac{3}{12}$$ *Write equivalent fractions.*

Since we cannot subtract $\frac{3}{12}$ from $\frac{2}{12}$, we need to borrow 1 from the 4, making it a 3. Since we have $\frac{2}{12}$, we need to write 1 as an equivalent fraction of $\frac{12}{12}$, so they have a common denominator. The we can add $\frac{2}{12}$ and $\frac{12}{12}$.

$$\overset{3}{\cancel{4}}\frac{2}{12} + \frac{12}{12} = 3\frac{14}{12}$$

$$\begin{array}{r} -2\frac{3}{12} \\ \hline \end{array} \qquad \begin{array}{r} -2\frac{3}{12} \\ \hline 1\frac{11}{12} \end{array}$$

Now we can subtract.

Therefore, $4\frac{1}{6} - 2\frac{1}{4} = \boxed{1\frac{11}{12}}$.

Case 2: Change the mixed numbers to improper fractions and use the rules given for adding and subtracting fractions. If you use this method you will not need to borrow when subtracting.

Example 1: Add: $3\frac{2}{5} + 2\frac{2}{3}$.

Solution:

$$\begin{array}{rcrcr} 3\frac{2}{5} & = & \frac{17}{5} & = & \frac{51}{15} \\ +2\frac{2}{3} & = & +\frac{8}{3} & = & +\frac{40}{15} \\ \hline & & & & \frac{91}{15} = 6\frac{1}{15} \end{array}$$

Change to improper fractions.

Find a common denominator and write equivalent fractions.

Add numerators, write sum over the common denominator.

Write answer as a mixed number.

Therefore, $3\frac{2}{5} + 2\frac{2}{3} = \frac{91}{15} = \boxed{6\frac{1}{15}}$.

Example 2: Subtract: $4\frac{1}{6} - 2\frac{1}{4}$.

Solution:

$$\begin{array}{rcrcr} 4\frac{1}{6} & = & \frac{25}{6} & = & \frac{50}{12} \\ {}^{-}2\frac{1}{4} & = & \frac{{}^{-}9}{4} & = & \frac{{}^{-}27}{12} \\ \hline & & & & \frac{23}{12} = 1\frac{11}{12} \end{array}$$

Change to improper fractions.

Find a common denominator and write equivalent fractions.

Subtract numerators, write difference over the denominator.

Write answer as a mixed number.

Therefore, $4\frac{1}{6} - 2\frac{1}{4} = \frac{23}{12} = \boxed{1\frac{11}{12}}$.

****

Now You Try (Section 9.2)

1) Add: $4\frac{1}{5} + 3\frac{1}{2}$

2) Subtract: $12\frac{3}{5} - 5\frac{1}{2}$

3) Add: $7\frac{5}{8} + 4\frac{1}{6}$

4) Subtract: $7\frac{1}{3} - 2\frac{3}{5}$

(Answers to **Now You Try** (Section 9.2) *are found on page 30.)*

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Multiplying and Dividing Mixed Numbers:

Change all mixed numbers to improper fractions and use the rules given for multiplying and dividing fractions.

Example 1: Multiply: $2\frac{2}{3} \cdot 1\frac{1}{2}$.

Solution: $2\frac{2}{3}\cdot 1\frac{1}{2}=\frac{8}{3}\cdot\frac{3}{2}$ *Change to improper fractions.*

$=\frac{\cancel{8}^{4}}{\cancel{3}_{1}}\cdot\frac{\cancel{3}^{1}}{\cancel{2}_{1}}$ *Divide out common factors between any numerator and any denominator.*

$=\frac{4}{1}=4$ *Multiply numerators, then multiply denominators.*

Therefore, $2\frac{2}{3}\cdot 1\frac{1}{2}=\boxed{4}$.

Example 2: Divide: $6\frac{1}{3}\div 4\frac{2}{9}$.

Solution: $6\frac{1}{3}\div 4\frac{2}{9}=\frac{19}{3}\div\frac{38}{9}$ *Change to improper fractions.*

$=\frac{19}{3}\cdot\frac{9}{38}$ *Multiply by the reciprocal.*

$=\frac{\cancel{19}^{1}}{\cancel{3}_{1}}\cdot\frac{\cancel{9}^{3}}{\cancel{38}_{2}}$ *Divide out common factors between any numerator and any denominator.*

$=\frac{3}{2}=1\frac{1}{2}$ *Multiply numerators, then multiply denominators.*

Write as a mixed number.

Therefore, $6\frac{1}{3}\div 4\frac{2}{9}=\boxed{1\frac{1}{2}}$.

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Now You Try (Section 9.3)

1) Multiply: $2\frac{3}{4}\times 5\frac{3}{8}$

2) Divide: $\frac{3}{5}\div 2\frac{3}{4}$

3) Multiply: $6\frac{3}{7}\times 2\frac{1}{3}$

4) Divide: $7\frac{1}{9}\div 4\frac{2}{3}$

(Answers to **Now You Try** (Section 9.3) *are found on page 30.)*

Exercises for Fractions

Do all the exercises on separate paper showing all your work.

1. Name the numerator of each fraction.

   a) $\frac{1}{5}$ b) $\frac{7}{15}$ c) $\frac{a}{b}$

2. Name the denominator of each fraction.

   a) $\frac{3}{5}$ b) $\frac{b}{17}$ c) 7

3. For the set of numbers $\left\{ \frac{3}{4}, \frac{7}{9}, \frac{12}{5}, \frac{9}{10}, \frac{2}{4}, \frac{20}{5} \right\}$, list all the proper fractions.

4. For the set of numbers $\left\{ \frac{1}{8}, \frac{2}{8}, \frac{9}{8}, \frac{3}{5}, \frac{14}{3} \right\}$, list all the improper fractions.

5. Using the model stated, show the fractions given below .

   a) Number line model $\frac{5}{6}$ b) Area model $\frac{3}{7}$

6. The shaded dots have a value of $\frac{2}{5}$. Draw a circle around "1".

   • • • 0 0
   • • 0 0 0
   • • 0 0 0
   • 0 0 0 0
   0 0 0 0 0

7. The shaded dots have a value of 2. Draw a circle around "1".

   • • • • 0
   • • • 0 0
   • • • 0 0
   0 0 0 0 0

8. The shaded dots have a value of $\frac{3}{7}$. Draw a circle around "1".

•	•	0	0
•	•	0	0
•	0	0	0
•	0	0	0
0	0	0	0

9. Circle the larger of the two fractions and name the fraction that is exactly half-way between the two.

a) $\frac{2}{3}$ $\frac{3}{4}$ b) $\frac{4}{7}$ $\frac{6}{11}$ c) $\frac{9}{13}$ $\frac{5}{7}$

10. For each lettered point on the number line below, express its position by a corresponding fraction.

a)
0 1 2 3
|——|——|——|——|——|——|——|——|——|
A B C

b)
0 1
|—|—|—|—|—|—|—|—|—|
D E F

c)
-1 0 2
|—|—|—|—|—|—|—|—|—|
G H I

11. Write 4 fractions equivalent to $\frac{5}{7}$.

12. Find a replacement value for x.

a) $\frac{x}{9} = \frac{5}{3}$ b) $\frac{8}{x} = \frac{4}{16}$

13. Simplify the following fractions.

a) $\frac{4}{14}$ b) $\frac{14}{35}$ c) $\frac{24}{48}$ d) $\frac{144}{196}$

e) $\frac{82}{104}$ f) $\frac{325}{725}$

14. Evaluate the following:

a) $\frac{3}{7}+\frac{1}{9}$ b) $\frac{3}{7}-\frac{1}{9}$ c) $\frac{3}{7}\cdot\frac{1}{9}$ d) $\frac{3}{7}\div\frac{1}{9}$

e) $\frac{1}{9}-\frac{3}{7}$ f) $\frac{1}{9}\div\frac{3}{7}$

What is the relationship between the answers to b and e?

What is the relationship between the answers to d and f?

15. Evaluate the following:

a) $\frac{19}{42}+\frac{13}{70}$ b) $\frac{3}{14}+\frac{6}{7}$ c) $\frac{4}{9}-\frac{2}{15}$ d) $\frac{16}{27}-\frac{2}{45}$

e) $\frac{4}{9}\times\frac{2}{3}$ f) $\frac{1}{3}\times\frac{3}{5}$ g) $\frac{4}{5}\times\frac{4}{7}$ h) $\frac{21}{34}\times\frac{8}{9}$

i) $\frac{3}{2}\div\frac{1}{5}$ j) $\frac{3}{7}\div\frac{2}{5}$ k) $\frac{5}{3}\div\frac{1}{2}$

16. What fraction must be added to $\frac{2}{5}$ to get a sum of $\frac{1}{2}$?

17. $\frac{2}{5}$ must be subtracted from what fraction to get a difference of $\frac{1}{2}$?

18. What fraction must be subtracted from $\frac{3}{5}$ to get a difference of $\frac{1}{2}$?

19. What fraction must be multiplied by $\frac{3}{5}$ to get a product of $\frac{1}{2}$?

20. What fraction must be divided into $\frac{2}{5}$ to get a quotient of $\frac{1}{2}$?

21. $\frac{2}{5}$ must be divided into what fraction to get a quotient of $\frac{1}{2}$?

22. Add: $3\frac{2}{5}+5\frac{3}{8}$

23. Subtract: $5\frac{3}{8}-3\frac{2}{5}$

24. Multiply: $3\frac{2}{5}\ x\ 5\frac{3}{8}$

25. Divide: $3\frac{2}{5}\div 5\frac{3}{8}$

26. Divide: $5\frac{3}{8}\div 3\frac{2}{5}$

27. Subtract: $7\frac{1}{5}-4\frac{9}{10}$

28. What number must be added to $7\frac{1}{6}$ to get a total of $19\frac{3}{4}$?

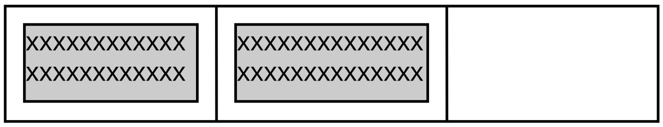
29. What number must be subtracted from $17\frac{2}{3}$ to get a difference of $6\frac{1}{2}$?

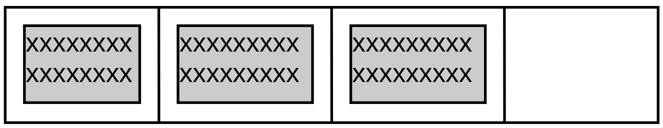
30. What number must be multiplied by $2\frac{5}{9}$ to get a product of $7\frac{1}{4}$?

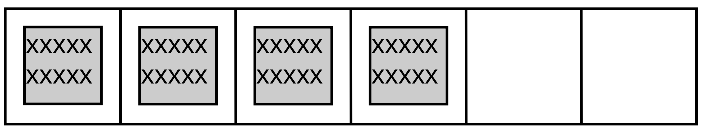
31. What number must be divided into $21\frac{1}{3}$ to get a quotient of $25\frac{3}{5}$?

Answers to <u>Now You Try</u>

Section 2.1:

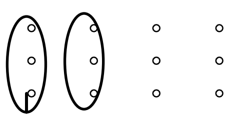
1) 

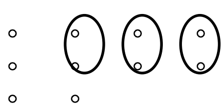
2) 

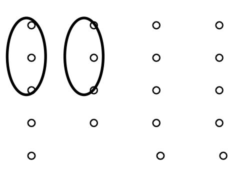
3) 

4) 1) $\frac{2}{3}$ 2) $\frac{3}{4}$ 3) $\frac{4}{6}$

Section 2.2:

1) 

2) 

3) 

4) 1) $\frac{2}{4}$ 2) $\frac{3}{5}$ 3) $\frac{2}{7}$

Section 2.3:

1) circle around 7 dots

2) circle around 20 dots

Section 2.4:

1) A = $\frac{1}{5}$ B = $\frac{2}{5}$ C = $1\frac{2}{5}$ D = $2\frac{3}{5}$

2) A = $\frac{2}{3}$ B = $1\frac{3}{6} = 1\frac{1}{2}$

3) Possible approximations: A $\approx \frac{1}{3}$ *(needs to be close to 0)* B $\approx 1\frac{5}{6}$ *(needs to be close to 2)*

Section 3.1:

1) $\frac{7}{8} = \frac{42}{48}$

2) $\frac{24}{40} = \frac{3}{5}$

Section 3.2:

1) $\frac{7}{10} \neq \frac{9}{14}$

2) $\frac{3}{4} = \frac{24}{32}$

Section 4.1:

1) $\frac{4}{7}$ is greater

2) $\frac{13}{20}$ is greater

Section 4.2:

1) $\frac{8}{14} = \frac{4}{7} \; or \; \frac{9}{14}$

2) $\frac{15}{20} = \frac{3}{4}$

Section 5:

1) $\frac{3}{4}$

2) $\frac{4}{7}$

Section 6:

1) $\frac{31}{40}$ 2) $\frac{17}{18}$ 3) $\frac{1}{6}$ 4) $\frac{7}{24}$

Section 7:

1) $\frac{5}{3}$ 2) $\frac{4}{63}$ 3) $\frac{6}{25}$ 4) $\frac{2}{5}$

Section 8:

1) $\frac{1}{10}$ 2) $\frac{4}{5}$ 3) $\frac{2}{27}$

Section 9.1:

1) a) $4\frac{5}{9}$ b) $25\frac{2}{21}$

2) a) $\frac{59}{7}$ b) $\frac{113}{9}$

Section 9.2:

1) $\frac{77}{10} = 7\frac{7}{10}$ 2) $\frac{71}{10} = 7\frac{1}{10}$

3) $\frac{283}{24} = 11\frac{19}{24}$ 4) $\frac{71}{15} = 4\frac{11}{15}$

Section 9.3:

1) $\frac{473}{32} = 14\frac{25}{32}$ 2) $\frac{12}{55}$ 3) 15 4) $\frac{32}{21} = 1\frac{11}{21}$